Human Development Research Paper 2010/47

Uncertainty and Sensitivity Analysis of the Human Development Index

Clara García Aguña
and Milorad Kovacevic

United Nations Development Programme
Human Development Reports
Research Paper

February 2011

Human Development
Research Paper
2010/47

Uncertainty and Sensitivity Analysis of the Human Development Index

Clara García Aguña
and Milorad Kovacevic

United Nations Development Programme
Human Development Reports
Research Paper 2010/47
February 2011

Uncertainty and Sensitivity Analysis of the Human Development Index

Clara García Aguña

and Milorad Kovacevic

Clara García Aguña is Statistics Consultant at the Human Development Report Office of the United Nations Development Programme. E-mail: clara.aguna@undpaffiliates.org.

Milorad Kovacevic is Head of Statistics at the Human Development Report Office of the United Nations Development Programme. E-mail: milorad.kovacevic@undp.org.

Comments should be addressed by email to the authors.

Abstract

This paper discusses the methodological judgments made during the development of the Human Development Index (HDI), and analyzes the quantitative and qualitative impact of different methodological choices on the HDI scores, as well as on the associated changes in ranking. This analysis is particularly pertinent this year, in light of the methodological refinements that have been implemented with the occasion of the HDI 20th anniversary.

Keywords: Index Numbers and Aggregation; Methodological Issues; Statistical Simulation Methods; Human Development.

JEL classification: C43, C15, C18, O15.

1 Introduction

The Human Development Index (HDI) was created in 1990, as an acknowledgment that income levels are not enough to capture the concept of human development. Under that premise, the HDI operationalized the broad concept of human development by combining health, education and income into a composite index. This paper analyzes the robustness of this measure, by assessing the quantitative and qualitative impact of different methodological choices both on the HDI scores and on the associated changes in ranking.

First, we review the methodological judgements and choices that need to be made to build a composite measure, i.e how to normalize the dimensions, the assignment of weights, and the aggregation method used to build the composite index. Second, we perform an uncertainty and sensitivity analysis, which includes all the decisions that can not be justified neither by theoretical reasons, nor by the data properties. The results of our simulations show that the HDI is robust to alternative methodological choices.

On top of this, two additional scenarios are considered: one considers data measurement error in the indicators, while the other challenges the current weighting scheme, by allowing each country to select its own optimal weights instead of applying a common set of weights to all countries. In both cases the impact on the HDI is very limited. Next, we detect a negative relationship between the HDI and the variability of its underlying indicators, which highlights the role of reducing gaps in performance between indicators in order to increase human development levels. Section 7 concludes by studying the "natural" way of grouping countries together by their human development level. A clustering analysis reveals a country classification which is broadly in line with the current HDI quartile classification.

2 Brief review of the HDI methodological choices

The HDI is a composite index which intends to capture the idea of human development by focusing on three dimensions: a long and healthy life, knowledge and a decent standard of living. Four indicators have been selected to measure these concepts: life expectancy at birth, mean years of schooling, expected years of schooling, and Gross National Income (GNI) per capita. This section reviews the methodological choices made to combine these indicators into the HDI.

2.1 Normalization

Given that the indicators used to measure achievement in each dimension are expressed in different units (years and dollars per capita), a normalization to a common scale is required. The methods that are most frequently used are standardization (or z-scores) and rescaling[1].

- Standardization: $\frac{x_i - mean(x)}{std(x)}$

  This method converts the indicators to a common scale of mean zero and standard deviation of one. Therefore it rewards exceptional behavior, i.e. above-average performance of a given indicator yields higher scores than consistent average scores across all indicators. This does not fit well our theoretical framework, since human development is a general concept where no dimension can be neglected in favor of another. For example: a poor performance in education cannot be fully compensated by an im-

[1] For a review of other normalization techniques, please see Nardo et al (2005)

provement in life expectancy. Therefore we will refrain from using this method, since human development is a multidimensional concept where balance in all dimensions should be rewarded.

- Re-scaling: $\frac{x_i - min(x)}{max(x) - min(x)}$

  This approach is easier to communicate to the public, given that it normalizes indicators to lie within an identical range [0,1]. A key advantage of this method over standardization is that re-scaling widens the range of the index, which is an advantage for those indicators lying within relatively small intervals. This is useful for the HDI to allow differentiation between countries with similar levels of achievement.

  However, this method is not appropriate in the presence of extreme values or outliers, which can distort the normalized indicator. To control for this, we need to take into consideration the distribution of our data. As can be seen in figure 1 there are no outliers for life expectancy, mean years of schooling and expected years of schooling, although GNI per capita does exhibit outliers. In order to avoid the impact of extreme values on the normalization procedure (figure 2), we transform the data using a logarithmic function. On top of this statistical justification, there is an economic reason behind this functional form, since increases in income are deemed to have diminishing marginal effects on human well-being. The Human Development approach assumes that income is not an end in itself but that is valued to the extent that extends people's capabilities to live meaningful lives, and that this occurs at a declining rate.

  As figure 2 shows, a log-transformation removes our concerns about extreme observations distorting the normalized indicator.

Figure 1: Distribution of the indicators for health and education

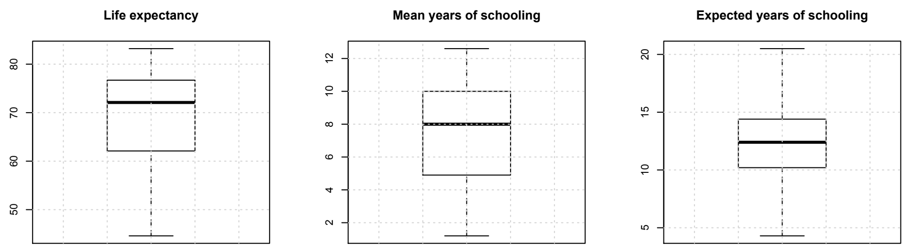
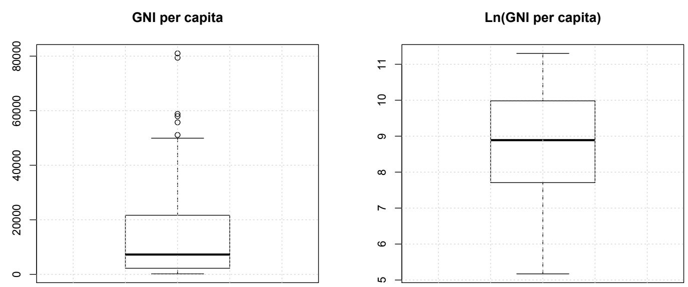


Figure 2: Effect of applying a logarithmic transformation on outliers



Therefore, given our theoretical framework and the properties of our data, re-scaling is deemed as the most adequate normalization method. However, the issue remains on how to select the maximum and minimum values needed for the re-scaling, which we will denote as "goalposts". Before considering the different alternatives, we will need to jump one step ahead for a moment, and consider the way the normalized indicators are aggregated into the HDI:

$$HDI = I_H^{w_H} \cdot I_E^{w_E} \cdot I_G^{w_G} = \left(\frac{x_{H_i} - min(x_H)}{max(x_H) - min(x_H)}\right)^{w_H} \cdot \left(\frac{x_{E_i} - min(x_E)}{max(x_E) - min(x_E)}\right)^{w_E} \cdot \left(\frac{ln(x_{G_i}) - ln(min(x_G))}{ln(max(x_G)) - ln(min(x_G))}\right)^{w_G}$$

where w = weight, H = health, E= education and G = GNI per capita.

The use of a geometric mean implies that the choice of the maximum value leaves the comparison between indicators unaffected. This is explained by the fact that the maximum only appears in the denominator (max-min), which is a constant, and therefore it does not affect the relative comparison. However, the choice of the minimum value will affect comparisons, so its value has to be carefully chosen.

The current approach is to set the minimum values to "natural zeros", ie subsistance levels. As the technical note 1 of the HDR 2010 states: "The minimum values are set at 20 years for life expectancy, at 0 years for both education variables and at $163 for per capita GNI. The life expectancy minimum is based on long-run historical evidence from Maddison (2010) and Riley (2005). Societies can subsist without formal education, justifying the education minimum. A basic level of income is necessary to ensure survival: $163 is the lowest value attained by any country in recorded history (in Zimbabwe in 2008) and corresponds to less than 45 cents a day, just over a third of the World Bank $1.25 a day poverty line."

Regarding maximum values, they could be set to the observed maximum,although using moving goalposts has the disadvantage of inter-temporal comparison. To deal with this issue fixed goalposts are used in the HDI 2010 methodology, namely the maximum values observed over the period for which the time series of the HDI is presented (1980-2010).[2]

Since HDI values are sensitive to the goalposts' choice, this will be one of the input factors in the sensitivity anaysis that we will perform in section 3.

[2]For a detailed review of the different HDI goalposts chosen over time, please refer to Kovacevic (2010).

2.2 Weighting

2.2.1 Explicit weights

The HDI attaches equal weights to each dimension of human development (health, education and living standard), on the grounds that they are all worth the same. This is subject to debate and will be therefore explored in our sensitivity analysis. However, it should be noted that in addition to the explicit weights attached to the dimensions, some dimensions may be implicitly granted more importance than others, due to, among others, the underlying distributions of the indicators or the normalization method.

2.2.2 Implicit weights

Power of differentiation Not all the dimension indices display the same level of differentiation between countries, which is defined in terms of the index range divided by the total number of countries. The income index features higher differentiation than the education index, which in turn performs better than the life expectancy index[3]. This seems to imply that differentiation implied by the income index is the most significant driver of differences in the HDI. It is worth noting that due to convergence in education and health, one can expect that the power of differentiation by indices will decrease over time.

[3] Power of differentiation values in 2010: income index: 0.0056; life expectancy index: 0.0036; education index: 0.0049.

Marginal weights The marginal weights are derived by calculating the elasticity of the HDI with respect to a one percent increase in any indicator. Here, the HDI elasticity expresses the sensitivity of the HDI to changes in an input indicator, keeping others unchanged.

As discussed before, the minimum goalpost affects relative comparisons across countries. However, mean years of schooling and expected years of schooling have their minimum values set to zero, what solves this caveat. Therefore, when we analyze the effect in the HDI of a change in the education indicators, the marginal effects are constant across countries. A 1% increase in either mean years of schooling or expected years of schooling yields about 0.16% increase in the HDI.

Regarding life expectancy, the minimum goalpost is different from zero and therefore precludes the marginal effects from being constant. A 1% increase in life expectancy yields an average 0.47% increase in the HDI, with a standard deviation of 0.03%, resulting in a coefficient of variation of 6.4%. The highest valuation of longevity is 1.38 times higher than the lowest. Afghanistan attached the highest valuation to longevity: 0.60% increase in the HDI due to a 1% increase in life expectancy. On the other side of the spectrum, Japan attaches the lowest valuation: 0.44% HDI increase given a 1% increase in life expectancy.

The same applies to income, where we cannot consider zero as the minimum goalpost since some income is needed for survival [4]. Therefore the marginal effects are not constant across countries. A 1% increase in GNI per capita yields an average 0.1% increase in the HDI, with a standard deviation of 0.04%, resulting in a coefficient of variation of 40%. Therefore, the differences across countries are more pronounced than in the case of life expectancy.

[4] See section 2.1.

The highest valuation of income comes from Zimbabwe, with a 4.1% increase in the HDI for each 1% increase in income. On the other hand, Liechtenstein attaches the lowest valuation: 0.05% increase in the HDI for each 1% increase in income. The large differences are explained by the fact that a number of countries in our sample are very close to the income subsistence level[5], and if they fall below that threshold they are unlikely to survive. This increases the impact of income in these countries. Moreover, this is compounded with the logarithmic transformation, which considers that income is translated into capabilities at a higher pace for low income levels. If we exclude from the study the countries that present extreme income observations [6], the following picture emerges: the country with the higher valuation of income is Guinea Bissau (whose income per capita is USD 538); a 0.28% increase in HDI for every 1% increase in income. This causes that the highest valuation of income is 5.2 times higher than the lowest.

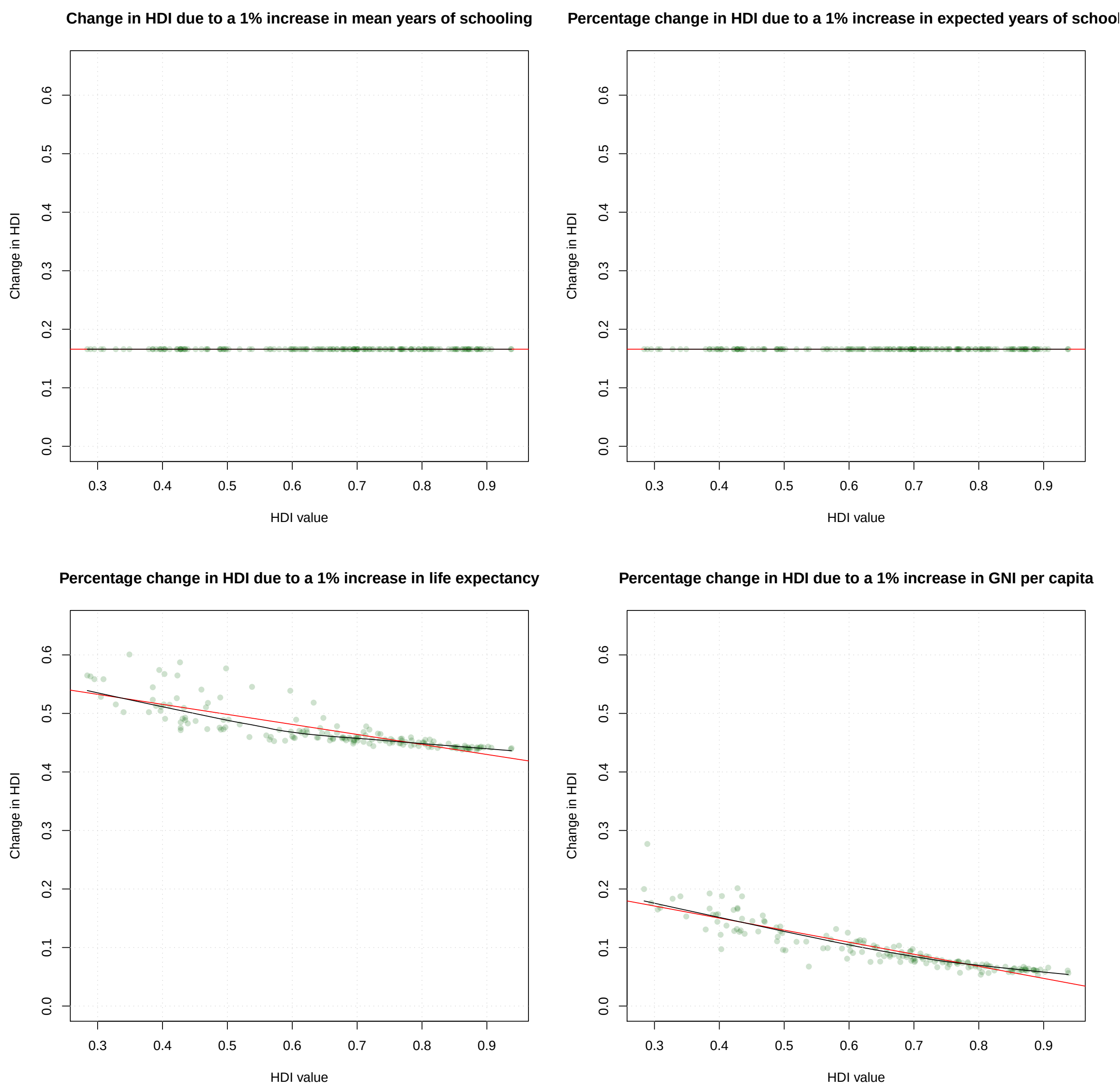
Graph 3 shows the marginal effects for all countries: [7]

[5]It might be argued that this is not the case reality, but rather than the GNI is not capturing their true standard of living, given that it does not consider subsistence agriculture, home production, and other unreported economic activities

[6]Burundi, Central African Republic, Democratic Republic of the Congo, Liberia, Niger, Sierra Leone and Zimbabwe.

[7]Note that outliers are excluded from the graph (Burundi, Central African Republic, Democratic Republic of the Congo, Liberia, Niger, Sierra Leone and Zimbabwe). The graph including all countries can be found in the Appendix.

Figure 3: Marginal weights.



Collinearity Notice that if two indicators are collinear and measure the same dimension, this dimension will be given an implicit higher weight, even though in principle each dimension has attached equal weights. The education dimension is measured by two indicators which are highly correlated (0.82) but are not collinear.

Although all the HDI indicators are positively correlated, this is not a symptom of double-counting, or apparent redundancy since they all represent different dimensions of development, as the factor analysis performed in Kovacevic (2010) proves.

Sections 3 and 4 analyze the consequences of different weighting schemes.

2.3 Aggregation

The most popular methods of aggregation in the literature are the arithmetic and geometric means. Although the arithmetic mean has been traditionally used to compute the HDI, the current HDI methodology applies a geometric mean to reflect the following trade-offs between dimensions:

1. Imperfect substitutability, ie poor performance in one dimension cannot be fully compensated by good performance in another, as it was the case with the arithmetic mean.
2. Reward of balance: one of the properties of the geometric mean is that it penalizes the differences in value between indicators[16], ie it rewards balanced achievement in all dimensions.
3. Higher impact of poor performance: the HDRO considers that the lower the performance in a particular dimension, the more urgent it becomes to improve achievements

in that dimension. By definition, the concavity of the geometric mean ensures that a reduction due to a decline in performance has a greater effect than an increase of the same magnitude[15].

Moreover, the arithmetic mean made the trade-off between life expectancy and years of education difficult to compare: although both variables are measured in years, the life expectancy range is wider, assigning a smaller implicit weight. Therefore if we use a linear function to aggregate the variables, we would implicitly agree that one year of education contributes more than a year of life expectancy.

3 Uncertainty and sensitivity analysis

As it has been discussed, a number of methodological choices have been made in order to construct the HDI. This section will assess the uncertainty of the index attributed to those judgements which can not be justified neither by theoretical reasons, nor by the data properties, namely, the functional form of life expectancy, i.e. log transformed or not, the minimum goalposts attached to life expectancy and income, and the weights assigned to the index dimensions. For the two education indicators we keep equal weights.

Following the uncertainty analysis, we will study which proportion of the total uncertainty can be attributed to each of the methodological choices. This is the sensitivity analysis. Using the uncertainty and sensitivity analysis we would like to check whether the HDI provides a biased picture of the countries' performance, and to what extent the different choice of input factors affect the countries' ranks compared to the original HDI ranks.

3.1 Definition of the input factors

Section 2.1 discussed why the most appropriate normalization method for the HDI indicators is deemed to be re-scaling. This method requires that each indicator has attached goalposts. Given that the choice of the maximum values does not affect relative comparisons, they will be excluded from the uncertainty exercise. In turn, we will focus on the range of plausible minimum values. Setting up the lower bound to the observed minimum is a compelling decision, although it distorts the index when the value of the variable approaches the mininum[8]. Although it is straightforward to set up "natural" zeros for mean years of schooling and expected years of schooling, the choice of 20 years as the subsistence value for life expectancy is less unambiguous. We will study the impact of assigning a range to the minimum goalpost: from 20 to the actual minimum observed, 44.6. We will reduce the actual minimum value observed by 5% to avoid 0 score on the component index. For the income component we study the impact of assigning the minimum goalpost from \$1 to the observed minimum of \$163.

The uncertainty analysis will analyze the impact of simultaneously considering different weights for the dimensions, and a range of plausible minimum values for the health and income dimension.

[8]Variables are normalized according to the formula $\frac{x_i - min(x)}{max(x) - min(x)}$. Thus, when x_i approaches the minimum, the numerator tends to zero, which is not compatible with the geometric aggregation method where all indices need to be positive.

The input factors are defined as follows:

1. We draw from the uniform distribution, $X_1 \sim U[0,1]$, to decide about the functional form of life expectancy to be used, i.e. if X_1 is lower than 0.5, then we apply logarithms to life expectancy; otherwise the life expectancy data remains unchanged.

2. Next, we draw from the uniform distribution to decide about the minimum goalposts for rescaling life expectancy and income. In the case of life expectancy, if $X_2 \sim U[0,1]$ is lower than 0.5 then we set the minimum goalpost to 20, the current value; otherwise, the minimum goalpost is drawn from a uniform distribution bounded between 20 and the observed minimum, reduced by 5% to avoid a score of 0, i.e. $X_3 \sim U(20, 0.95*$(obs min)). Regarding income, if $X_4 \sim U[0,1]$ is lower than 0.5 then we set the minimum goalpost to ln(163), which is the current value. Otherwise we assign any value in the range ln(1) to ln(163), ie $X_5 \sim U(log(1), log(163))$.

3. We simulate the dimension weights by drawing from an uniform distribution, i.e. $X_i \sim U(0.1, 1)$ $\forall i = 6, 7, 8$, where X_6: weight for health, X_7: weight for income and X_8: weight for education.

   We have chosen 0.1 as the minimum possible weight, not to exclude any dimension from the overall index. Note that in the next stage the weights will be normalized, that is, $w_6 = \frac{x_6}{x_6+x_7+x_8}$, $w_4 = \frac{x_7}{x_6+x_7+x_8}$, and $w_6 = \frac{x_8}{x_6+x_7+x_8}$, so we don't need to be concerned about the upper bound.

3.2 Monte Carlo simulations

Given the assumed distributions of the input factors, we generate 5000 random draws of $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8\}$, and for each we compute the following output:

1. Index value[9]: $Y_c = I_1^{w_1} \cdot I_2^{w_2} \cdot I_3^{w_3}$, where 1= health, 2 = education and 3 = income.
2. Y_c Ranking
3. Differences in the HDI value between country A and B: $D_{AB} = Y_A - Y_B$
4. Average absolute shift in ranking: $\bar{R} = \frac{1}{N} \sum_{c=1}^{n} |rank_{ref}(Y_c) - rank(Y_c)|$

3.3 Sensitivity indices

The sensitivity analysis aims at attributing to each of the input factors (minimum goalposts, functional forms and weights) a share of the total output uncertainty. Following Saltelli and al (2004), we will use variance-based methods to assess the output uncertainty. This approach has a number of advantages, First, they are independent of the model used. This is very relevant in our case, given the non-linearity of the model used to compute the HDI[10]. Moreover, the variance decomposition methods will allow us to evaluate the effect of a factor while all others are varying as well.[11]. An additional benefit is the flexibility it gives to define the uncertainty factors, which can be described in terms of their probability density

[9]The weights have been previously normalized.

[10]Recall that to normalize the dimensions' indices we need to set up a minimum goalpost, what makes the model non-linear.

[11]This would not be the case if we were using a perturbative approach, where we allow one factor to vary while all the others are kept constant.

function.

The rationale of the method is as follows: the total output variance $V(Y)$ can be decomposed into:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{123456}$$

where:

$$V_i = V_{X_i}\{E_{X_{-i}}(Y|X_i)\}$$

$$V_{ij} = V_{X_i X_j}\{E_{X_{-ij}}(Y|X_i, X_j)\} - V_{X_i}\{E_{X_{-i}}(Y|X_i)\} - V_{X_j}\{E_{X-j}(Y|X_j)\}, \text{ etc.}$$

$x_i, i = 1, ..., 6$ denote the input factors. $E_{X_{-i}}$ is the expectation (integral) over all factors except X_i, whereas the variance V_{X_i} is the variance over X_i and its marginal distribution.

The main effects are measured by the so-called first order sensitivity index, which indicates the relative importance of an individual input variable X_i in driving the total uncertainty:

$$S_i = \frac{V_i}{V(Y)}$$

The total effects are measured by the amount of output variance that would remain unexplained if X_i, and only X_i, were left free to carry over its uncertainty range, all the other variables having been fixed:

$S_{T_i} = \frac{V(Y) - V_{X_{-i}} E[V(Y|X_{-i})]}{V_i}$, where $V_{X_{-i}}$ is the variance calculated over all factors but X_i.

A low value of S_{T_i} indicates that the input factor X_i is irrelevant in the analysis of the uncertainty (typically, a value below 0.3 is considered a low value). The difference $S_{T_i} - S_i$ is a measure of how much X_i is involved in interactions with any other input variable.

3.4 Results

3.4.1 HDI values

Figure 4 shows the distribution of the HDI simulations, based on 1000 draws of $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8\}$, compared with the original HDI value, which is denoted by a cross in the graph. The HDI lies within the $30^{th} - 55^{th}$ percentile of the distribution[12]. This implies that the HDI provides a robust measure that is not biased by neither the goalposts, nor the weights used.

3.4.2 HDI differences

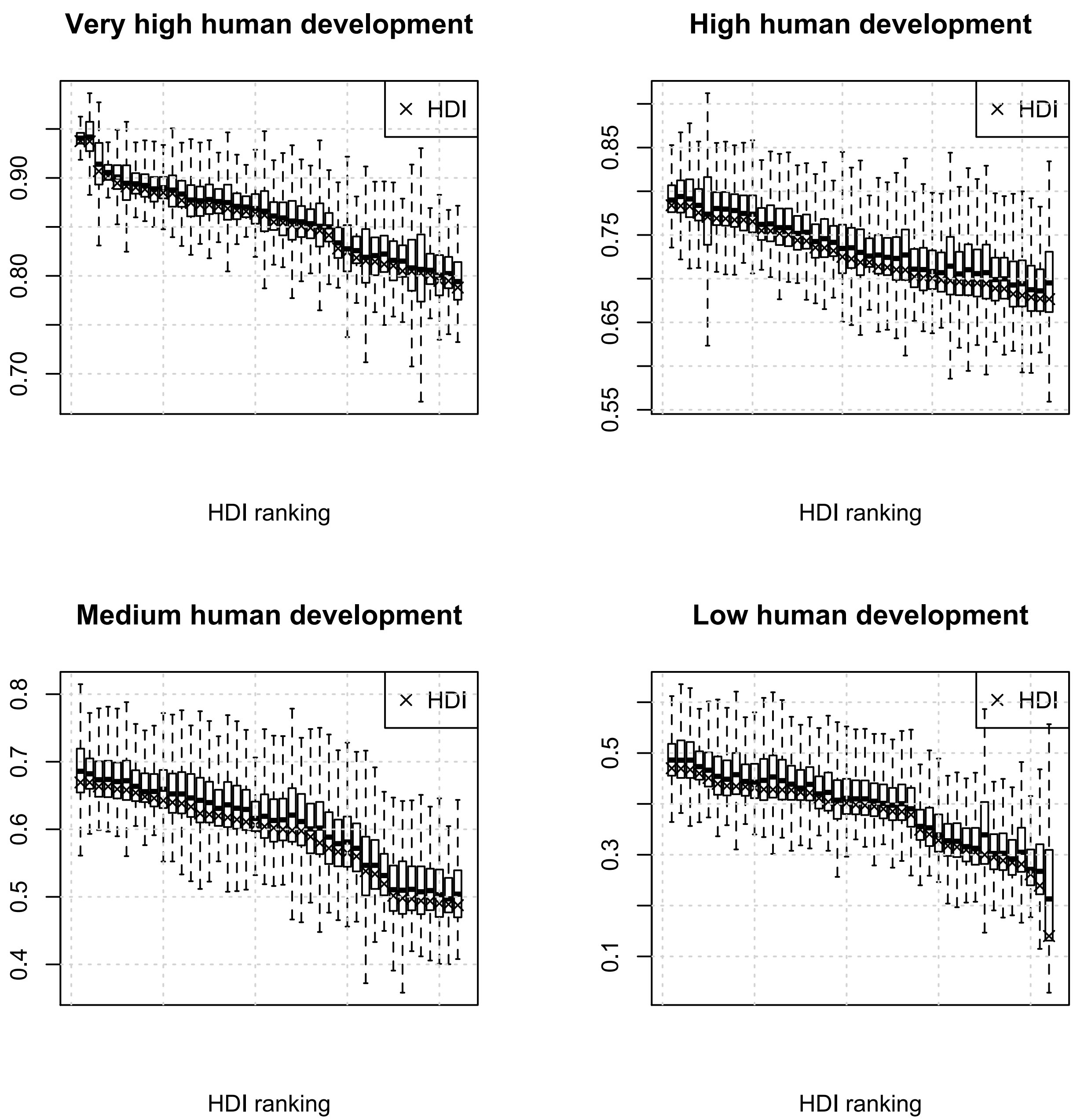
This section studies the impact of the uncertainty inputs on the difference in HDI score between countries, what will allow us to assess if the HDI values of two countries are statistically significantly different at 1% significance (p-value < 0.01)[13]. The results for the top-ten countries in the HDI 2010 ranking can be found in table 1[14], where "Yes" means that the null hypothesis of equal means is rejected.

[12] The average absolute percentage change between the HDI and the median is -2.1%,with a standard deviation of 2.7%.

[13] We set up a t-test on the series of the differences and test the null hypothesis of mean zero.

[14] The detailed results for all countries can be found in the annex.

Figure 4: Uncertainty analysis of the HDI values by quartile



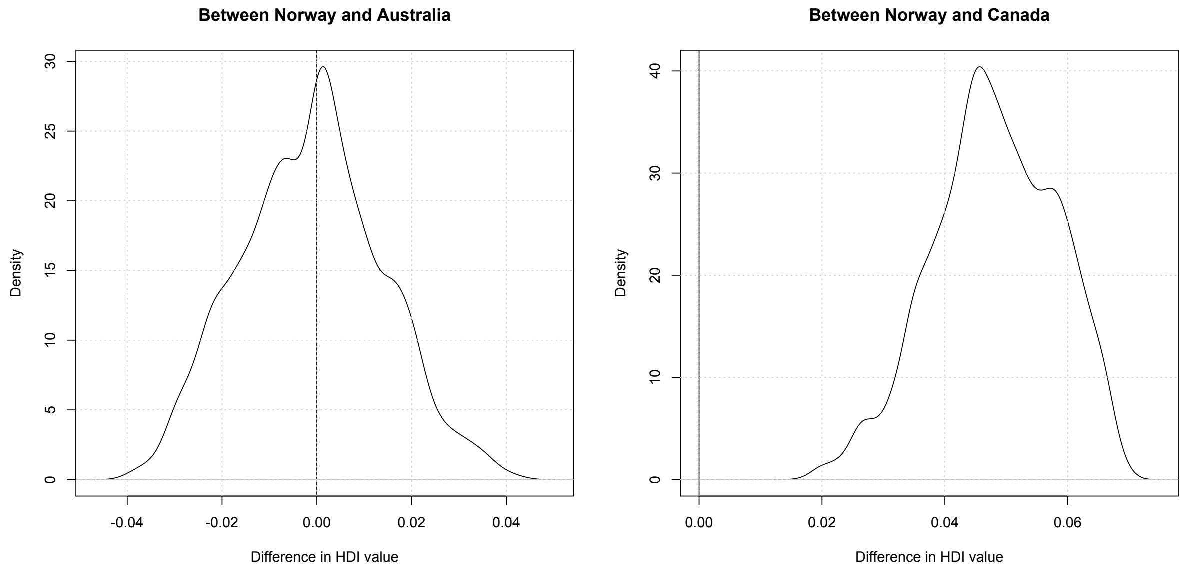
From Table 1 we see that among the top-ten countries there is a low statistically significant differentiation. Norway and Australia are significantly different form 6 countries each, although they are not significantly different between themselves. Liechtenstein is the least different from other countries followed by New Zealand.

Table 1: Statistically different HDI values

	Norway	Australia	New.Zealand	United.States	Ireland	Liechtenstein	Netherlands	Canada	Sweden
Norway									
Australia	No								
New Zealand	No	Yes							
United States	Yes	No	No						
Ireland	Yes	Yes	No	No					
Liechtenstein	No	No	No	No	No				
Netherlands	Yes	Yes	No	Yes	No	No			
Canada	Yes	Yes	No	No	No	No	No		
Sweden	Yes	Yes	No	No	No	No	No	No	
Germany	Yes	Yes	No	Yes	No	No	No	No	No

If we analyze which are the factors driving the differences in score between Norway and Australia, the results of the sensitivity analysis indicate that the trigger for the functional form of the life expectancy index does not play a role in the uncertainty analysis, and the same applies to the triggers for the minimum goalposts: no matter whether the current goalposts for income and life expectancy are used, or whether we allow them to vary within the uncertainty range, since its impact on the HDI is minimal. Note that these results hold regardless of the values taken by the other input factors, which are moving freely in their uncertainty range.

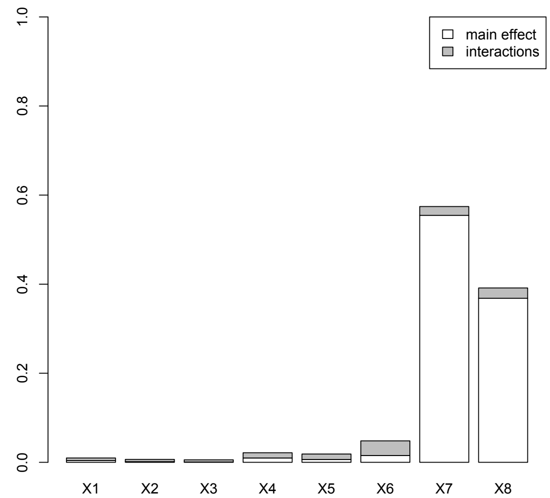
Figure 5: Simulated HDI value differences



The sensitivity analysis identifies the weights attached to income and education as the main sources of uncertainty (see figure 7). This result is not surprising: GNI per capita is over USD 58,800 in Norway, while it refers to USD 38,700 in the case of Australia. On the other hand, expected years of schooling particularly favor Australia, 20.5 years, as opposed to Norway, 17.3 years[15]. Therefore, what is driving the differences in HDI value is whether the weight for income is higher than the one for education, or viceversa. Section 4 will revisit this issue, by studying the case where countries are given the opportunity of selecting the weights that maximize their scores, and how this affects the overall HDI values and rankings.

[15]Mean years of schooling are roughly the same for both countries: 12.6 in Norway and 12.0 in Australia. Same applies to life expectancy: 81 years in Norway, and 81.9 n Australia

Figure 6: Sensitivity analysis of the differences between Norway and Australia

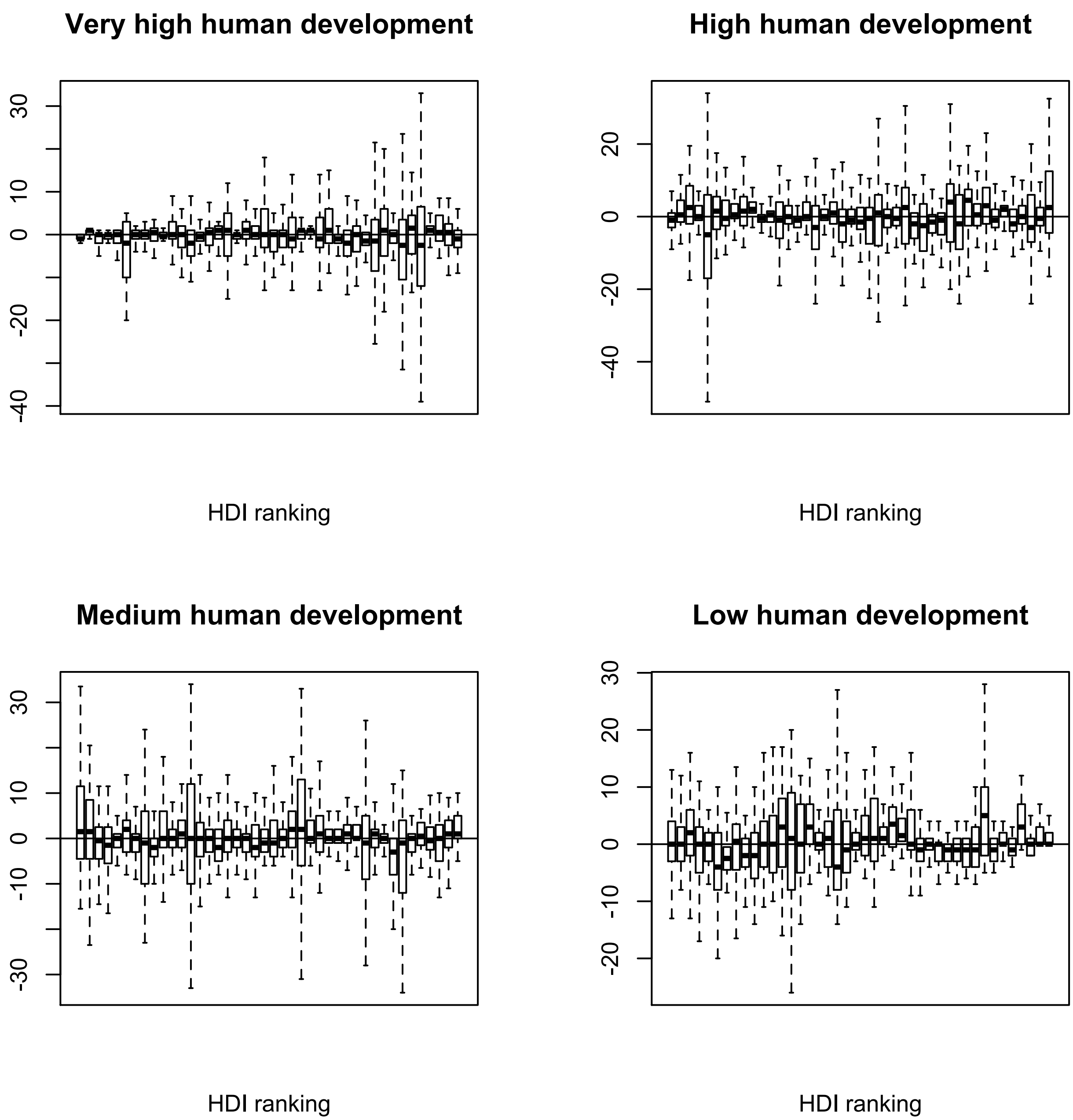


3.4.3 HDI rankings

Now we will proceed to assess how the uncertainty in the functional form of life expectancy, the minimum goalposts for rescaling of life expectancy and income and the weights affect the HDI ranking, both at the individual country level, and at the average absolute shift level.

Figure 7 displays the distribution of the rankings derived from applying different weights, normalizations and an alternative functional form of life expectancy. The median ranking, denoted by the segment highlighted on bold, derived from the simulations is extremely close to the original HDI ranking. The rectangles represent the $[25^{th}, 50^{th}]$ percentiles of the distributions.

Figure 7: Uncertainty analysis of the HDI rankings by quartile



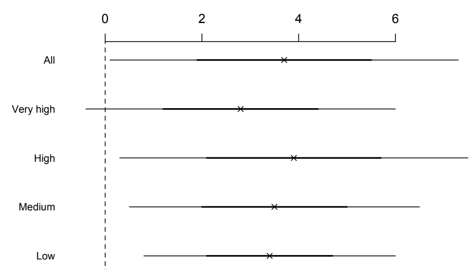
The graph seems to indicate that the impact differs by level of human development. In order to assess this hypothesis, we consider the average absolute shift in the ranking across countries, which is defined as:

$$\bar{R} = \frac{1}{N}\sum_{c=1}^{N} |rank_{HDI}(Y_c) - rank_{sim}(Y_c)|$$

where N = number of countries; sim= simulation

As figure 8 shows[16], the shifts in ranking are relatively minor and seem not to compromise the robustness of the HDI, regardless of the development level.

Figure 8: Average absolute change in ranking, by development level

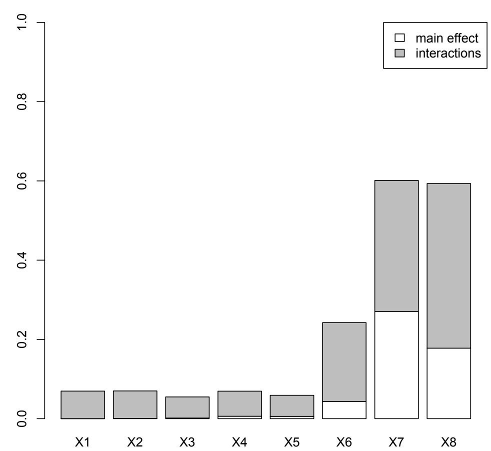


From the sensitivity analysis of this measure we conclude that the weights applied to income and education are the main drivers of uncertainty, both at the individual level and at the interaction level, as figure 9 illustrates.It is worth noting that the functional form of life expectancy and the choices related to the normalization method have a negligible impact on the HDI average shift in ranking. The weight attached to the health dimension has little importance, although its influence increases when we consider the effect of its interactions with other variables. The fact that the weights applied to income and education are the

[16]A table with the underlying figures can be found in the Appendix, see table 6

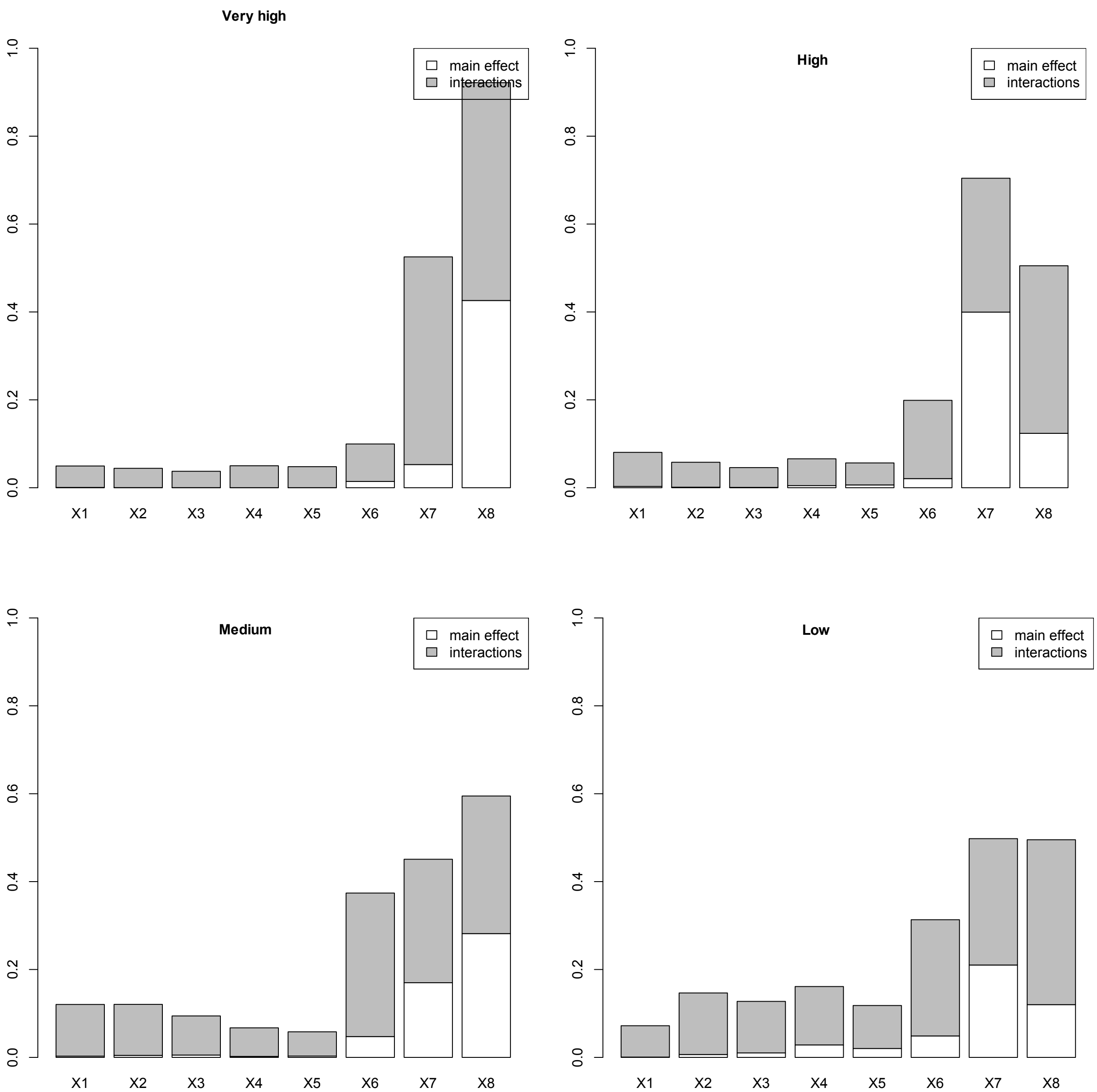
main drivers of the (little) uncertainty in the average ranking shift, seems to be in line with the higher power of differentiation between countries of these indicators.

Figure 9: Sensitivity indices of the average shift in the rank across all countries



The sensitivity analysis by development group is broadly in line with the sensitivity analysis when all countries are considered: the input factors driving the variance in the shift in ranking are the dimension's weights, particularly the income and education dimensions. The weight of the education dimension is particularly important for the very high development group, both as a single factor, and taking its interactions into account. That is likely to be driven by the large differentiation in expected years of schooling, the highest among all development groups. Regarding the high development countries, the main driver is income, while for the medium development group the main driver is education but health has the highest interaction effects, what seems natural given that this block has the highest differentiation in life expectancy of all groups. Concerning the low development group, as expected the interaction of life expectancy plays a major role.

Figure 10: Sensitivity indices from the average absolute rank shift, by human development level



4 Optimal weights

The HDI assigns a common weighting scheme to all countries. This implies that each dimension is valued equally across countries, i.e. education, health and income are deemed to have the same importance in achieving human development, regardless of the country under consideration. This may lead to question how fair the weights are, i.e. whether certain countries are particularly favored by the fixed set of weights, given the different data characteristics across countries. This section analyzes this issue by allowing countries to select their own, most favorable weights. The exercise is set up as follows: we perform an optimization exercise where the objective function aims at maximizing the HDI score. To ensure that no dimension is excluded, we will subject the weights to upper and lower bounds. On top of this, three scenarios are considered according to the allowed degree of dominance across dimensions:

$max \quad w_1 \cdot ln(I_1) + w_2 \cdot ln(I_2) + w_3 \cdot ln(I_3)$[17]

subject to:

1. Case 1: High dominance

$$\begin{cases} 0.05 \leq w_i \quad i = 1, 2, 3 \\ w_1 + w_2 + w_3 = 1 \end{cases}$$

   Here we allow one dimension to have its weight as high as 0.9.

[17]Note that the original aggregation, $I_1^{w_1} \cdot I_2^{w_2} \cdot I_3^{w_3}$, has been transformed using logarithms in order to convert it to a linear programming problem

2. Case 2: Medium dominance
$$\begin{cases} 0.1 \leq w_i \leq 0.6 \quad i = 1,2,3 \\ \\ w_1 + w_2 + w_3 = 1 \end{cases}$$
In this case, we allow one dimension to have its weight as high as 0.6

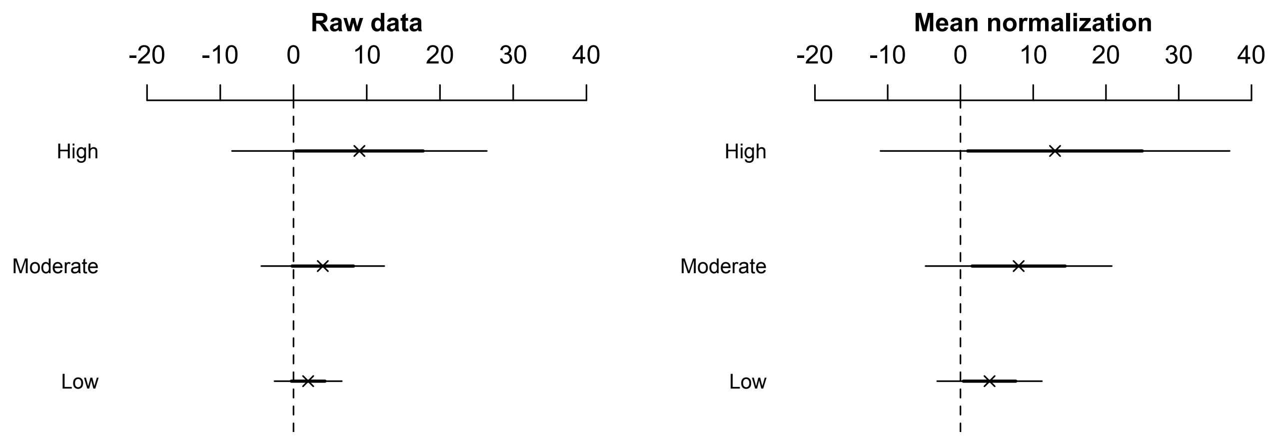
3. Case 3: Low dominance
$$\begin{cases} 0.2 \leq w_i \leq 0.47 \quad i = 1,2,3 \\ \\ w_1 + w_2 + w_3 = 1 \end{cases}$$
The highest weight allowed is 0.47.

In all cases 1 refers to education, 2 to health and 3 to income.

The median change in ranking[18] across countries is displayed in figure 11, where the line indicates the 95% confidence interval and the segment highlighted in bold denotes ± 1 standard deviation. The exercise is performed over the raw data; as well as over the data divided by the dimension mean across countries, in order to account for relative comparative advantage. As expected, the more extreme weighting schemes yield higher differences in terms of HDI values and rankings. The results in terms shifts in HDI ranking are heterogenous across countries, what is reflected in the large standard deviation. Regardless of the scenario, allowing each country to select its own optimal weights yields does not allow us to reject the hypothesis that the average change in ranking is zero.

[18]In absolute terms

Figure 11: Median shift in rank; by level of weight dominance



5 HDI and the variability of its indicators

This section studies the relationship between the HDI score of a given country and the variability of its three underlying dimensions, ie what is the relationship, if any, between the HDI score and a balanced performance in health, education and income. While the HDI values provide a quantitative indication of trends in human development, changes in the dimension's variability convey information on the quality of the changes: an increase in human development may be achieved by improving the performance in specific dimensions, but also by reducing gaps in performance between indicators.

In order to measure the variability of the underlying dimensions we will calculate their coefficient of variation:$\frac{\sigma}{\mu}$. As can be seen in figure 12, countries with higher levels of human development exhibit less variability, since they tend to achieve high values in all the underlying dimensions. The opposite holds generally true for countries with lower levels of development, see the trend. The average variability in the very high development group is 0.11, in the high development group is 0.16, while in the medium and low are 0.26 and 0.39, respectively. This reflects the fact that countries with lower levels of development generally display larger discrepancies in performance between dimensions, and that focusing only in particular dimensions while allowing performance gaps between dimension yields only marginal results. However, it is worth noting that there is a certain variance in the results: although Nigeria and Afghanistan belong to the low development group, their variabilities are below the average variability of the very high development group. The same applies to a number of medium development countries (Gabon, Botswana, Namibia and Congo)

Given that the HDI aggregates its three dimensions by using a multiplicative structure, which rewards balance, it could be argued that this is driving the reverse association between the HDI value and the variability of its components. The coefficient of variation for a given country is independent of the method of aggregation, ie it is not affected by the perfect or imperfect substitutability of its components. However, this will affect the HDI value and therefore the classification of countries into different levels of development. To assess the extent of the multiplicative structure's influence in the relationship between the HDI and the variability of its components, we run the same exercise for the HDI derived using an additive functional form. The average coefficient of variation by development group remains virtually unchanged[19].

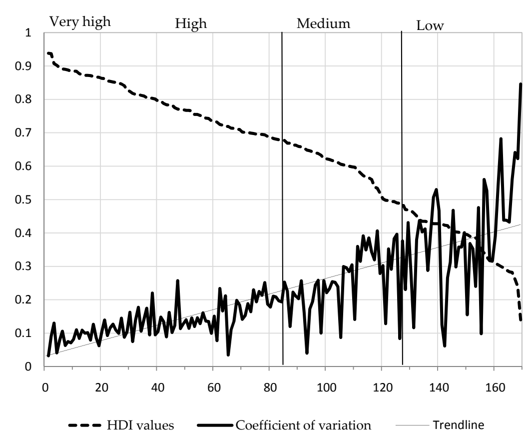
[19] See figure 19 in the Appendix

The Pearson correlation coefficient between the HDI and the coefficient of variation is -0.81, what reflects a high degree of negative association between the HDI and the variability of its dimensions. If we would be using an additive structure to aggregate the dimensions, the correlation would be somewhat lower: -0.76.

Table 2: Average coefficient of variation by human development group:

	Multiplicative	Additive
Very high	0.11	0.11
High	0.16	0.17
Medium	0.26	0.25
Low	0.39	0.39

Figure 12: HDI values and the variability of their underlying dimensions (ordered by the HDI ranking)



The same results apply if we consider the four underlying indicators instead of the dimensions (in which case the correlation between the HDI and the coefficient of variation is -0.80).

6 Measurement error of the raw data

This section analyzes the impact of data measurement error, which may affect the HDI in the form of, for example, ex-post revisions of their underlying indicators. This is particularly relevant given that the current methodology uses estimates for the current year[20], with the belief that improving the timeliness of the data enhances the relevance of the HDI. We address this issue by adding a normally distributed random error[21], with mean zero and a standard deviation 5% of the associated mean for each country[22]. In order to obtain robust results, 1000 simulations are performed.

Overall, the effect of accounting for measurement error is minor, with very high correlations between the HDI calculated from the original data for 2010, and the data with the added random error[23] both in terms of scores (Pearson correlation) and rankings (Spearman and Kendall). If we look at an aggregate measure of the overall change in ranking, the effects are moderate: the absolute mean shift in ranking is 3.3, with a standard deviation of 0.2.

[20]Life expectancy, mean years of schooling and expected years of schooling are estimated for 2010 by the data producer, while GNI has been estimated using the projections published in the IMF's World Economic Outlook.

[21]Note that measurement error can be decomposed into bias (systematic error) and variance (random error). This section refers exclusively to the random error, and leaves aside the issue of the systematic error, that may be gauged from the data revisions. Anecdotically, the revisions of the indicators used in the HDR 2007/2008 reveal that GDP seems to be more accurate than the other variables. However, it may be argued that in the current economic juncture, GNI estimates are more uncertain. On a related note, higher levels of human development seem to be linked to higher levels of accuracy, what may be due to the amount of resources available to build statistical capacity.

[22]Results for 10% can be found in the Appendix, figures 17 and 18

[23]Pearson: 0.99; Spearman: 0.95; Kendall: 0.99

Figure 13: Distribution of the HDI scores accounting for measurement error

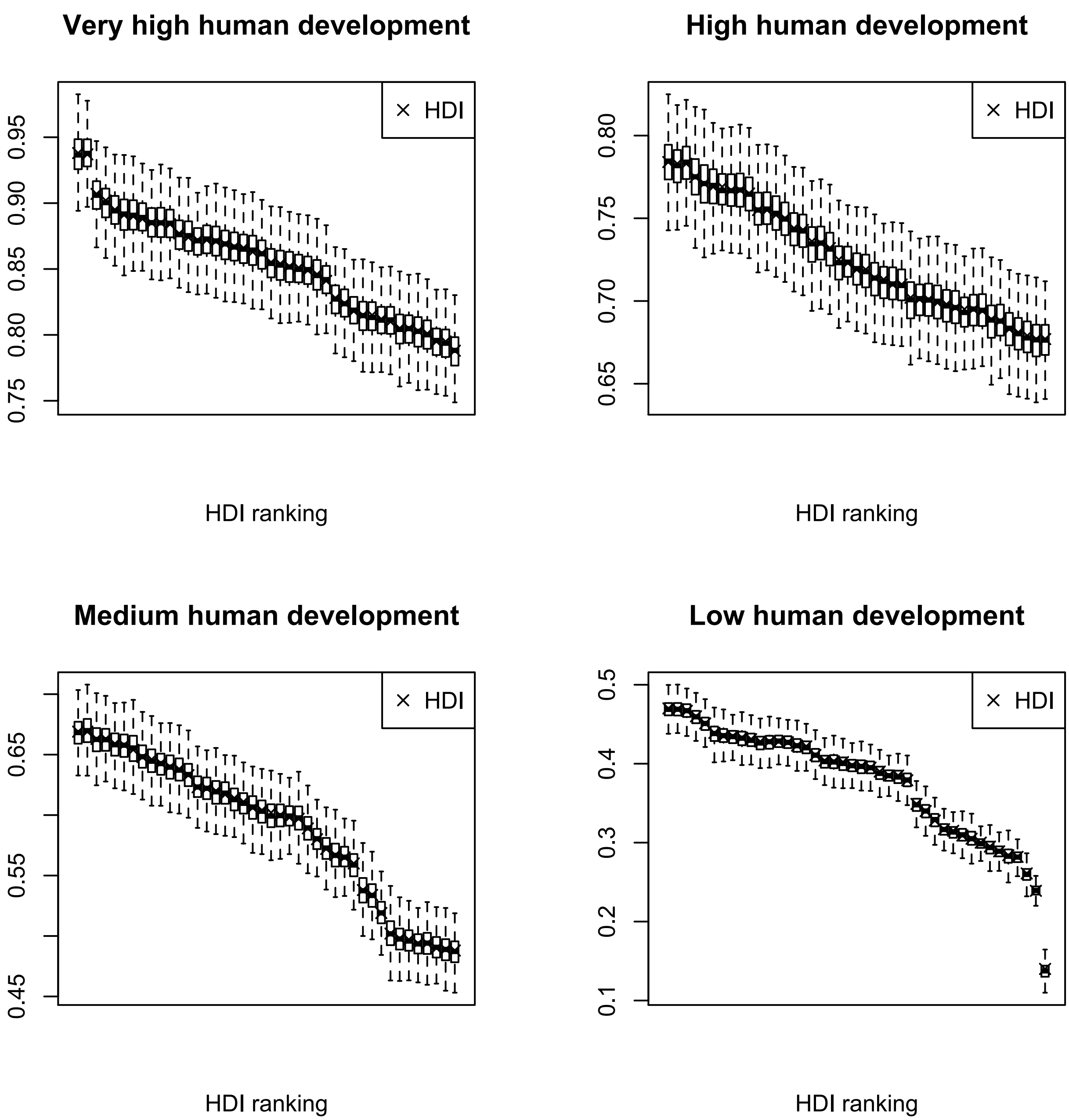
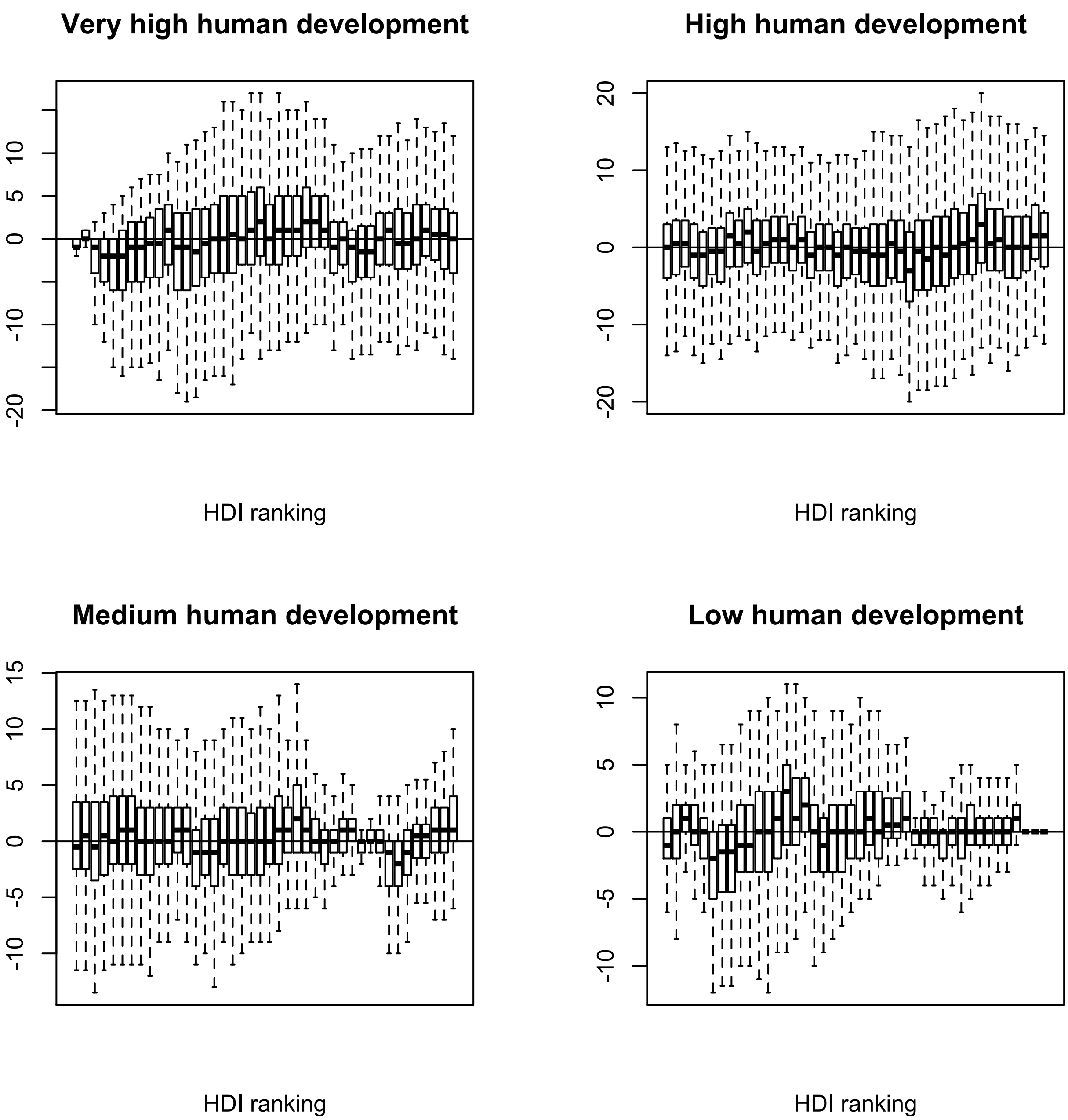


Figure 14: Distribution of the HDI ranking shifts accounting for measurement error



7 Towards a "natural" country classification by human development level

The Human Development Report Office classifies countries into four levels of human development, which correspond to the the quantiles of the HDI distribution[24]. Namely, these groups are: very high, high, medium and low human development. However, the choice of the number of groups is rather arbitrary. If we turn to other multilateral organizations for guidance on how many groups to select, their choices also seem to be of practical nature. For example, the World Bank classifies countries by income into five blocks: low-income, lower-middle-income, upper-middle-income, high-income and high-income OECD members.

In this section we aim at identifying natural groupings of human development. In order to do so, we will group together regions that are similarly situated with respect to the dimensions underlying the HDI, rather than with respect to the aggregated overall index[25] .

The analysis is performed using both hierarchical and non-hierarchical methods, which in our case yield similar results. This serves as a robustness check to ensure the validity of the analysis. In this section we refer only to hierarchical clustering, although the interested reader can refer to the Appendix for the non-hierarchical results.

[24]Before 2010, development groups were based on HDI values rather than quantiles. The new classification avoids using threshold HDI values, which may be seen as somewhat arbitrary, and it has reduced the amount of variation within each group: for example, the medium human development group ranged from 0.500 to 0.799 based on HDI values, while the range using quantiles is reduced to 0.488-0.669

[25]Additionally, we have performed the same analysis considering the indicators underlying the HDI, instead of the dimensions. The results remain broadly the same. Please see tables 8, 9, and 10 in the Appendix.

The clustering analysis relies on the principle the members of a cluster, e.g. countries in the same development group, are more similar to each other than to members of other clusters. To measure the distance between countries we will use the regular Euclidean distance, defined as:

$$\mathrm{d(x,\, y)} = \sqrt[2]{(x_{MYS} - y_{MYS})^2 + (x_{EYS} - y_{EYS})^2 + (x_{GNI} - y_{GNI})^2 + (x_{LE} - y_{LE})^2}$$

where:

x =country x; y =country y; MYS = mean years of schooling; EYS = expected years of schooling; GNI = gross national income; LE = life expectancy.

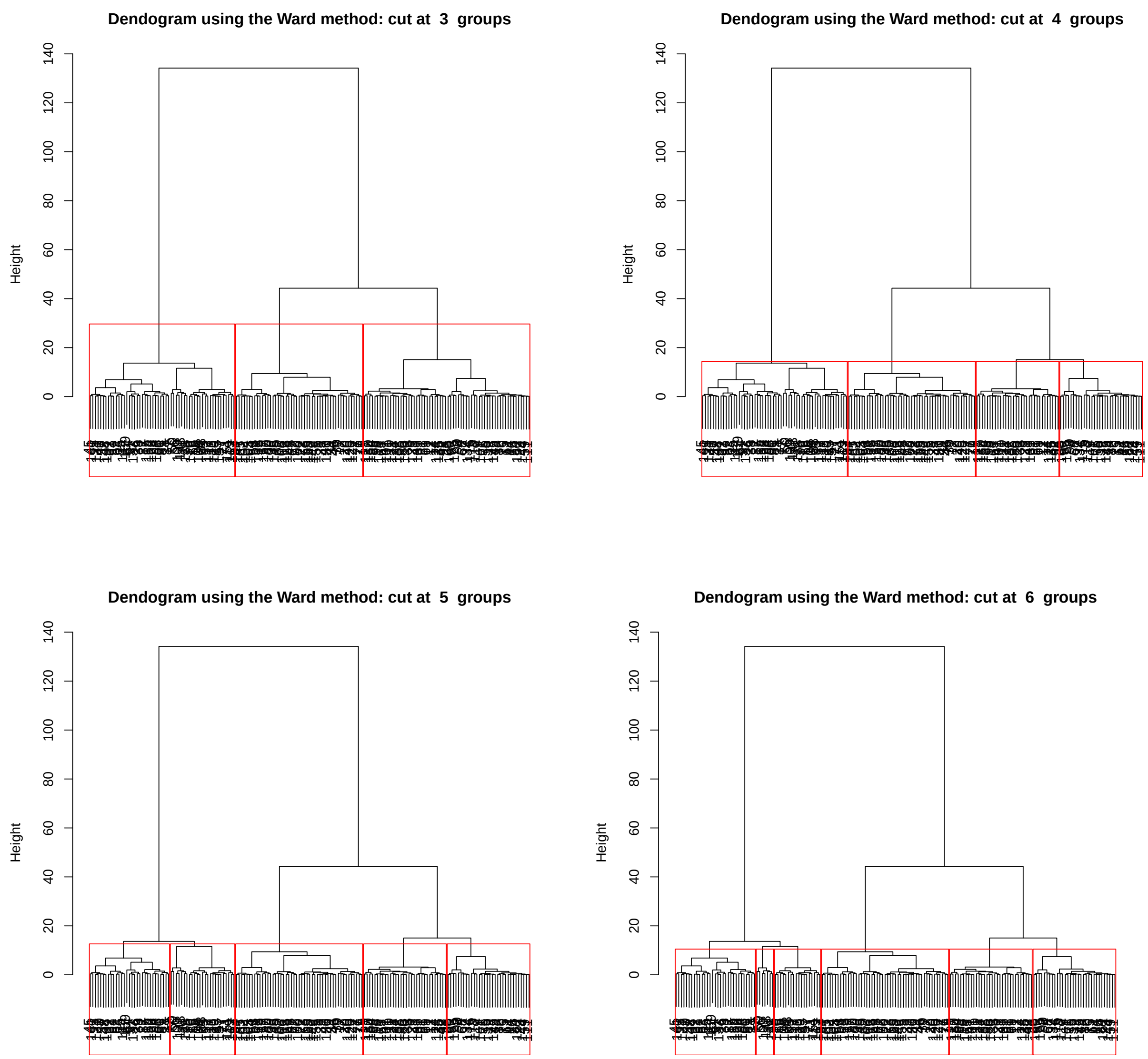
In order to measure the distance between clusters, several methods could be used. We have applied a number of the most popular choices in the literature, and discarded those methods that yield clusters with only a few members. Following this rule, the Ward method is deemed as the most suitable one[26].

As can be seen in the dendograms[27] in figure 15, the most balanced country clustering is the one obtained by considering four clusters. This yields a group whose HDI values range from 0.14 to 0.54, i.e. containing the low human development countries, plus the lower tier

[26]The results for other distances among clusters can be found in the annex.

[27]This term refers to the tree structure that shows how sample units are combined into clusters, the height of each branching point corresponding to the distance at which two clusters are joined.

Figure 15: Ward method: clusters yield at different cut-off points



of the medium human development group[28]. The countries that belong to the second cluster range from 0.56 to 0.72 in the HDI, i.e. they are countries in the second or third tier of the medium development group, plus the lower tier of the high development group. The third cluster groups together the remaining high human development countries plus the lower tier of the high development group. Cluster number four agglomerates the countries with a very high level of human development.

Table 3: Hierarchical clustering using the Ward method

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Afghanistan	1	0.35	Albania	3	0.72	Argentina	3	0.78	Andorra	4	0.82
Angola	1	0.4	Algeria	3	0.68	Azerbaijan	3	0.71	Australia	4	0.94
Bangladesh	1	0.47	Armenia	3	0.7	Bahamas	3	0.78	Austria	4	0.85
Benin	1	0.44	Belize	3	0.69	Bahrain	4	0.8	Belgium	4	0.87
Botswana	2	0.63	Bolivia	2	0.64	Barbados	4	0.79	Brunei Darussalam	4	0.8
Burkina Faso	1	0.3	Bosnia & Herzeg.	3	0.71	Belarus	3	0.73	Canada	4	0.89
Burundi	1	0.28	Brazil	3	0.7	Bulgaria	3	0.74	Denmark	4	0.87
Cambodia	2	0.49	Cape Verde	2	0.53	Chile	3	0.78	Finland	4	0.87
Cameroon	1	0.46	China	2	0.66	Croatia	3	0.77	France	4	0.87
Central African Rep.	1	0.32	Colombia	3	0.69	Cyprus	4	0.81	Germany	4	0.88
Chad	1	0.3	Costa Rica	3	0.72	Czech Republic	4	0.84	Greece	4	0.86
Comoros	1	0.43	Dominican Rep.	2	0.66	Estonia	4	0.81	Hong Kong, China	4	0.86
Congo	2	0.49	Ecuador	3	0.7	Hungary	4	0.8	Iceland	4	0.87
Congo (D.R.)	1	0.24	Egypt	2	0.62	Latvia	3	0.77	Ireland	4	0.9
Côte d'Ivoire	1	0.4	El Salvador	2	0.66	Libya	3	0.76	Israel	4	0.87
Djibouti	1	0.4	Fiji	2	0.67	Lithuania	3	0.78	Italy	4	0.85
Equatorial Guinea	2	0.54	Georgia	3	0.7	Malaysia	3	0.74	Japan	4	0.88
Ethiopia	1	0.33	Guatemala	2	0.56	Malta	4	0.82	Korea	4	0.88
Gabon	2	0.65	Guyana	2	0.61	Mexico	3	0.75	Kuwait	3	0.77
Gambia	1	0.39	Honduras	2	0.6	Montenegro	3	0.77	Liechtenstein	4	0.89

[28]According to the current classification into quartile groups, countries whose HDI 2010 lies between 0.14 and 0.47 belong to the low human development group, those ranging between 0.488 and 0.669 belong to the medium human development, while the high human development group lies between 0.677 and 0.784, and the very high human development group requires a value of 0.788 or higher.

[29]where DG stands for Development Group: 1 = low human development, 2 = medium human development, 3 = high human development, and 4 = very high human development. It refers to the development groups as of HDI 2010.

Table 3: Hierarchical clustering using the Ward method

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Ghana	1	0.47	Indonesia	2	0.6	Panama	3	0.76	Luxembourg	4	0.85
Guinea	1	0.34	Iran	3	0.7	Peru	3	0.72	Netherlands	4	0.89
Guinea-Bissau	1	0.29	Jamaica	3	0.69	Poland	4	0.8	New Zealand	4	0.91
Haiti	1	0.4	Jordan	3	0.68	Portugal	4	0.8	Norway	4	0.94
India	2	0.52	Kazakhstan	3	0.71	Romania	3	0.77	Qatar	4	0.8
Kenya	1	0.47	Kyrgyzstan	2	0.6	Russia	3	0.72	Singapore	4	0.85
Lao P.D.R.	2	0.5	Maldives	2	0.6	Saudi Arabia	3	0.75	Spain	4	0.86
Lesotho	1	0.43	Mauritius	3	0.7	Serbia	3	0.74	Sweden	4	0.88
Liberia	1	0.3	Micronesia	2	0.61	Slovakia	4	0.82	Switzerland	4	0.87
Madagascar	1	0.44	Moldova	2	0.62	Slovenia	4	0.83	U. A. E.	4	0.82
Malawi	1	0.38	Mongolia	2	0.62	Trinidad & Tobago	3	0.74	United Kingdom	4	0.85
Mali	1	0.31	Morocco	2	0.57	Uruguay	3	0.76	United States	4	0.9
Mauritania	1	0.43	Nicaragua	2	0.56						
Mozambique	1	0.28	Paraguay	2	0.64						
Myanmar	1	0.45	Philippines	2	0.64						
Namibia	2	0.61	Sri Lanka	2	0.66						
Nepal	1	0.43	Suriname	2	0.65						
Niger	1	0.26	Syria	2	0.59						
Nigeria	1	0.42	Tajikistan	2	0.58						
Pakistan	2	0.49	Thailand	2	0.65						
Papua New Guinea	1	0.43	Macedonia	3	0.7						
Rwanda	1	0.38	Tonga	3	0.68						
Sao Tome and Principe	2	0.49	Tunisia	3	0.68						
Senegal	1	0.41	Turkey	3	0.68						
Sierra Leone	1	0.32	Turkmenistan	2	0.67						
Solomon Islands	2	0.49	Ukraine	3	0.71						
South Africa	2	0.6	Uzbekistan	2	0.62						
Sudan	1	0.38	Venezuela	3	0.7						
Swaziland	2	0.5	Viet Nam	2	0.57						
Tanzania	1	0.4									
Timor-Leste	2	0.5									
Togo	1	0.43									
Uganda	1	0.42									
Yemen	1	0.44									
Zambia	1	0.4									
Zimbabwe	1	0.14									

8 Conclusion

This paper aimed at studying the sensitivity of the HDI to the methodological judgments and choices that were made during its construction, as well as to quantify the uncertainty in the HDI values and ranks based on these methodological choices. The analysis has confirmed that the uncertainty is unavoidable in composite indices – and the HDI is not an exception.

The results have shown that the HDI provides a robust measure that is not statistically significantly biased by neither the choice of the functional form of life expectancy, nor the minimum goalposts, nor by the weights attached to the HDI dimensions. At the same time, the sensitivity analysis has shown that the difference in HDI values between a number of countries is not statistically significant at 1% level, and that such similarity is not determined by the methodological choices. For example, New Zealand's HDI is statistically significantly different from Australia but not from other eight top-ranked countries including the top Norway. The sensitivity analysis has shown that the factors driving this result are the choice of weights for income and education, and that the minimum goalposts and their interactions with other considered sources of uncertainty have a rather negligible impact. The weight of the health dimension has little importance in this case.

Similar findings hold for the sensitivity analysis of rankings. Again the most important sources of uncertainty are the weights attached to income and education. The weight of the health dimension is more important for uncertainty of ranking through its interaction with the other factors. The importance of the weights used with income and education seems to be determined by the higher power of differentiation between countries.

We also explored some of the ideas embodied into the envelopment data analysis with respect to optimal weighting which we formulated as allowing each country to select a set of optimal weights for three dimensional indices so that the HDI is maximized providing that the weights satisfy certain constraints – they must lie between a given minimum and maximum and add up to 1. The test of a hypothesis that the median change in rank is equal to zero has shown that it cannot be rejected at 5% significance level. Thus, the current equal weighting has proven to be robust. This is a consequence of high correlation between the component indicators.

An additional analysis looked at the relationship between the HDI value and the variability of its component indices. We found that higher HDI values are associated with less variance in the underlying components, thus more balanced components. This is generally true for most of composite indices, but it seems to be enhanced by the geometric mean aggregation of the HDI. On top of this, we also explored the impact of data measurement error, assuming that the component indicators may be subject to random (measurement) errors. Our Monte Carlo analysis finds that the ranking is still robust.

Finally, we compared the classification of the countries into development groups according to HDI distribution quartiles to a "natural" classification based on cluster analysis using the component indicators, component indices, with and without an additional variable - the HDI. Several different association (distance) measures were used in the analysis. We found that the classification into four groups is the most balanced. The most stable groups are the very high developed and the low developed countries. The two middle human developed groups – high developed and middle developed are regrouped depending on the method used showing less stability and more uncertainty of the current classification according to distribution quartiles.

Overall, the sensitivity and uncertainty analysis have confirmed that the HDI is relatively robust index with the most sensitivity exhibited to the choice of weights for income and education component.

References

[1] Anand, S. and A. K. Sen (1994). Human Development Index: Methodology and measurement. Occasional Papers 12, Human Development Report Office, United Nations Development Programme.

[2] Nardo, M., Saisana, M., Saltelli, A., Tarantola, S., Hoffman, A., and Giovannini, E.. (2005) Handbook on Constructing Composite Indicators Methodology and User Guide. OECD Statistics Working Paper. STD/DOC(2005)3.

[3] Saisana, M., Saltelli and Tarantola, S. (2004). Uncertainty and sensitivity analysis techniques as tools for the quality assessment of composite indicators. Journal Royal Statistical Society.

[4] Kovacevic, M. (2010) Review of HDI critiques and improvements. Human Development Report Office Background Paper; United Nations Development Programme.

[5] Saisana, M. (2008) 2007 Composite Learning Index: Robustness Issues and Critical Assessment. JRC Scientific and Technical Reports. EUR 23274 EN 2008.

[6] Saisana, M., Saltelli, A. (2010) Uncertainty and Sensitivity Analysis of the 2010 Environmental Performance Index. JRC Scientific and Technical Reports. EUR 24269 EN 2010.

[7] UNDP (1990-2009): Human Development Report, New York and Oxford: United Nations Development Programme.

[8] Ravallion, M. (2010). Troubling Tradeoffs in the Human Development Index. World Bank Policy Research Working Paper 5484.

[9] Despotis, DK. A reassessment of the human development index via data envelopment analysis (2005). Journal of the Operational Research Society (2005) 56. 969-980

[10] Hoyland, B., Moene, K., and Willumsen, F. (2010). The Tyranny of International Index Rankings. *mimeo*.

[11] Wooldridge, J. Introductory Econometrics: A Modern Approach. (2002). South-Western College Publications. Second edition.

[12] Hsiao, C. Analysis of Panel Data (2003). Econometric Society Monographs No. 34. Cambridge University Press.

[13] Lebart, L., Morineau, A., Piron, M. (2000) Statistique Exploratoire Multidimensionnelle. Dunod, Paris. Third edition.

[14] Saltelli, A. (2002). Sensitivity Analysis for Importance Assessment. Risk Analysis, Vol. 22, No. 3, 2002.

[15] Let's talk Human Development HDRO blog. Discussions between Francisco Rodríguez and Martin Ravaillon on the Human Development Index.

[16] Herrero, C., Martinez, R., Villar, A. (2010). Improving the Measurement of Human Development. HDRO Background Paper.

[17] Singleton Jr., R., Straits, B. (2009). Approaches to Social Research. Oxford University Press, USA, 5^{th} edition.

9 Appendix

Figure 16: Marginal weights: all countries

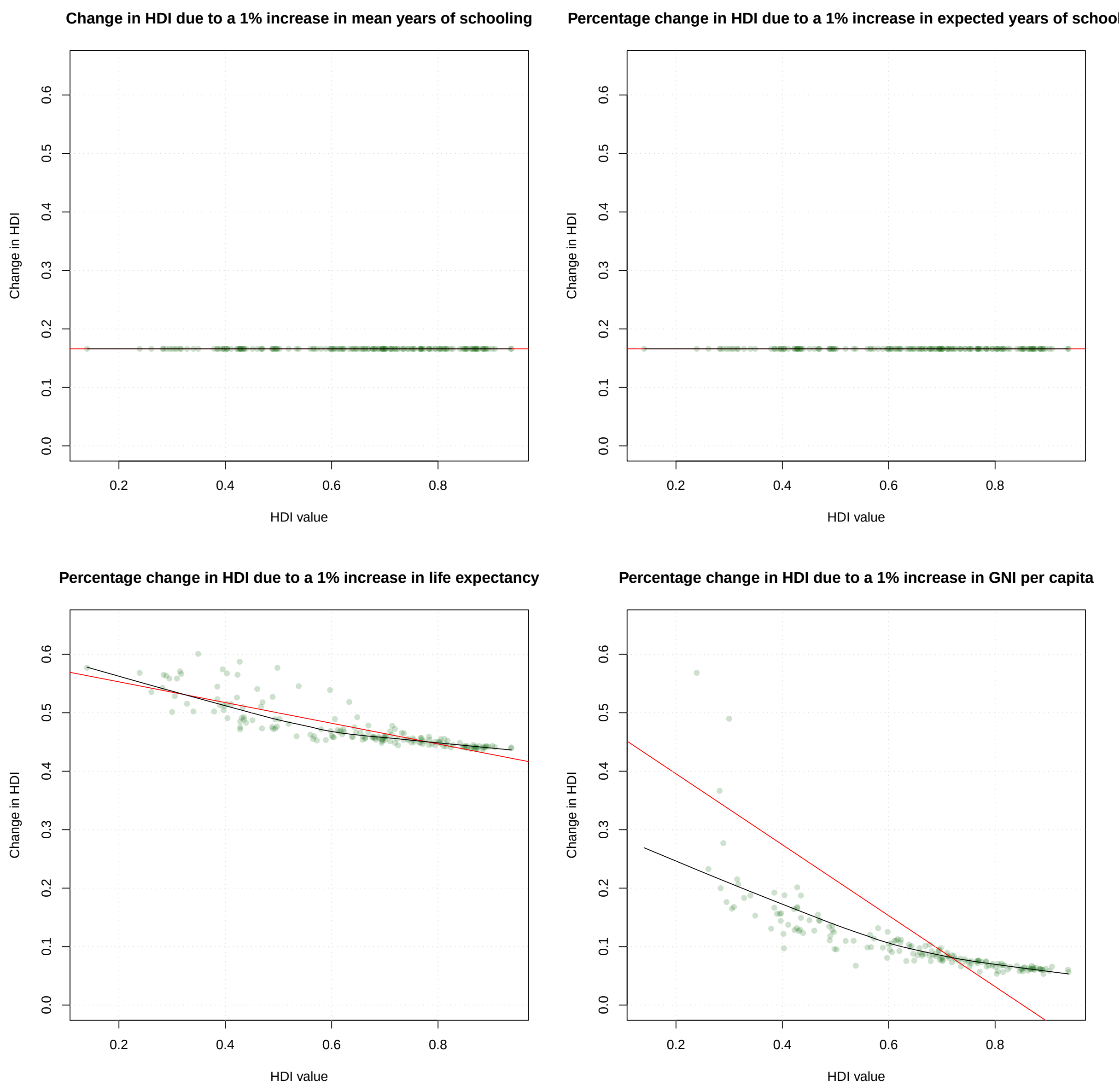


Table 4: Range of the indicators underlying the HDI

	LE	GNI	MYS	EYS
Very high	9.5	1.6	5.3	9.0
High	13.7	2.6	6.0	5.2
Medium	27.9	2.5	8.2	6.9
Low	22.9	3.3	6.0	6.7

Table 5: Sensitivity analysis of the differences in HDI value between Norway and Australia

Input factor	Main effect	Total effect	Interactions
X_1	0.00	0.01	0.01
X_2	0.00	0.01	0.01
X_{2a}	0.00	0.01	0.01
X_3	0.01	0.02	0.01
X_{3a}	0.01	0.02	0.01
X_4	0.01	0.05	0.03
X_5	0.55	0.57	0.02
X_6	0.37	0.39	0.02
Sum	*0.96*	*1.08*	*0.12*

Table 6: Detailed table on the average shift in ranking across countries

HDI ranking	Country	Median ranking	Range of rankings
1	Norway	-1	[-2,0]
2	Australia	1	[-7,1]
3	New Zealand	0	[-27,1]
4	United States	0	[-14,1]
5	Ireland	0	[-16,1]
6	Liechtenstein	-1	[-21,5]
7	Netherlands	0	[-8,2]
8	Canada	0	[-6,3]
9	Sweden	-1	[-6,3]
10	Germany	1	[-9,4]
11	Japan	0	[-8,10]
12	Korea (Republic of)	0	[-18,6]
13	Switzerland	-1	[-12,9]
14	France	-1	[-8,8]
15	Israel	1	[-15,6]
16	Finland	1	[-8,4]
17	Iceland	1	[-15,12]
18	Belgium	0	[-5,2]
19	Denmark	1	[-10,8]
20	Spain	0	[-8,8]
21	Hong Kong, China (SAR)	0	[-18,18]
22	Greece	0	[-10,5]
23	Italy	0	[-7,10]
24	Luxembourg	-1	[-18,19]
25	Austria	0	[-8,9]
26	United Kingdom	0	[-4,4]
27	Singapore	-1	[-21,21]
28	Czech Republic	1	[-9,16]
29	Slovenia	-1	[-6,2]
30	Andorra	-1	[-30,15]
31	Slovakia	0	[-14,8]
32	Malta	-2	[-38,27]
33	United Arab Emirates	-1	[-7,6]
34	Estonia	1	[-18,21]
35	Cyprus	-1	[-8,5]
36	Brunei Darussalam	0	[-16,17]
37	Hungary	-1	[-36,24]
38	Qatar	-1	[-51,35]
39	Bahrain	1	[-6,5]
40	Poland	0	[-14,8]
41	Portugal	1	[-5,9]
42	Barbados	-1	[-13,7]
43	Bahamas	-1	[-11,7]

Table 6: Detailed table on the average shift in ranking across countries

HDI ranking	Country	Median ranking	Range of rankings
44	Chile	1	[-22,20]
45	Lithuania	1	[-8,12]
46	Argentina	0	[-5,7]
47	Kuwait	-4	[-59,36]
48	Latvia	0	[-15,20]
49	Montenegro	0	[-10,14]
50	Croatia	1	[-11,19]
51	Romania	2	[-8,9]
52	Uruguay	2	[-3,12]
53	Libyan Arab Jamahiriya	-1	[-10,6]
54	Panama	1	[-5,7]
55	Saudi Arabia	-1	[-22,14]
56	Mexico	0	[-10,10]
57	Malaysia	-1	[-8,3]
58	Bulgaria	0	[-6,12]
59	Trinidad and Tobago	-2	[-29,16]
60	Serbia	0	[-7,7]
61	Belarus	1	[-29,14]
62	Costa Rica	-2	[-24,24]
63	Peru	-1	[-12,11]
64	Albania	-2	[-13,15]
65	Russian Federation	0	[-36,12]
66	Kazakhstan	0	[-42,30]
67	Azerbaijan	1	[-17,14]
68	Bosnia and Herzegovina	-1	[-9,11]
69	Ukraine	2	[-29,40]
70	Iran (Islamic Republic of)	-1	[-14,6]
71	Mauritius	-2	[-13,7]
72	The former Yugoslav Republic of Macedonia	-1	[-23,12]
73	Brazil	-1	[-14,5]
74	Georgia	3	[-21,36]
75	Venezuela (Bolivarian Republic of)	-1	[-25,14]
76	Armenia	2	[-17,20]
77	Ecuador	1	[-9,18]
78	Belize	2	[-14,27]
79	Colombia	0	[-9,9]
80	Jamaica	1	[-5,9]
81	Tunisia	-1	[-14,13]
82	Jordan	0	[-10,12]
83	Turkey	-2	[-24,21]
84	Algeria	-1	[-13,9]
85	Tonga	2	[-17,40]
86	Fiji	0	[-16,40]

Table 6: Detailed table on the average shift in ranking across countries

HDI ranking	Country	Median ranking	Range of rankings
87	Turkmenistan	1	[-25,25]
88	China	-2	[-18,11]
89	Dominican Republic	0	[-14,16]
90	El Salvador	0	[-10,6]
91	Sri Lanka	2	[-8,21]
92	Thailand	0	[-11,9]
93	Gabon	-1	[-26,24]
94	Suriname	-1	[-11,7]
95	Bolivia	0	[-17,30]
96	Paraguay	0	[-9,10]
97	Philippines	1	[-7,14]
98	Botswana	0	[-35,34]
99	Moldova (Republic of)	0	[-15,25]
100	Mongolia	0	[-11,21]
101	Egypt	-2	[-8,10]
102	Uzbekistan	0	[-14,26]
103	Micronesia (Federated States of)	0	[-11,14]
104	Guyana	-1	[-10,16]
105	Namibia	-2	[-17,10]
106	Honduras	-1	[-6,15]
107	Maldives	-1	[-8,18]
108	Indonesia	0	[-4,13]
109	Kyrgyzstan	2	[-13,34]
110	South Africa	1	[-36,35]
111	Syrian Arab Republic	0	[-7,29]
112	Tajikistan	1	[-12,32]
113	Viet Nam	0	[-5,28]
114	Morocco	0	[-11,17]
115	Nicaragua	1	[-7,23]
116	Guatemala	0	[-10,15]
117	Equatorial Guinea	-1	[-34,53]
118	Cape Verde	1	[-15,16]
119	India	0	[-8,3]
120	Timor-Leste	-3	[-23,12]
121	Swaziland	-2	[-37,16]
122	Lao People's Democratic Republic	0	[-9,5]
123	Cambodia	-1	[-10,9]
124	Solomon Islands	1	[-8,7]
125	Pakistan	1	[-15,11]
126	Congo	1	[-19,9]
127	Sao Tome and Principe	2	[-5,10]
128	Kenya	0	[-15,14]
129	Bangladesh	0	[-9,13]

Table 6: Detailed table on the average shift in ranking across countries

HDI ranking	Country	Median ranking	Range of rankings
130	Ghana	1	[-13,18]
131	Cameroon	0	[-20,12]
132	Myanmar	0	[-8,6]
133	Yemen	-3	[-22,10]
134	Benin	-3	[-10,6]
135	Madagascar	1	[-17,15]
136	Mauritania	-2	[-11,5]
137	Papua New Guinea	-2	[-16,11]
138	Comoros	-1	[-12,18]
139	Nepal	2	[-17,18]
140	Togo	1	[-10,17]
141	Lesotho	0	[-26,20]
142	Nigeria	0	[-14,13]
143	Uganda	3	[-7,16]
144	Senegal	1	[-7,7]
145	Haiti	1	[-9,14]
146	Angola	-4	[-14,27]
147	Djibouti	0	[-14,17]
148	Tanzania (United Republic of)	0	[-3,9]
149	Cote d'Ivoire	1	[-7,14]
150	Zambia	1	[-11,18]
151	Gambia	1	[-3,9]
152	Malawi	1	[-3,12]
153	Rwanda	3	[-5,14]
154	Sudan	0	[-9,21]
155	Afghanistan	-1	[-14,7]
156	Guinea	0	[-7,18]
157	Ethiopia	0	[-7,10]
158	Sierra Leone	-1	[-5,2]
159	Central African Republic	-1	[-7,3]
160	Mali	-1	[-6,5]
161	Burkina Faso	0	[-7,10]
162	Liberia	4	[-5,29]
163	Chad	-1	[-5,5]
164	Guinea-Bissau	0	[-3,5]
165	Mozambique	-1	[-4,4]
166	Burundi	3	[-1,13]
167	Niger	0	[-2,13]
168	Congo (Democratic Republic of the)	0	[-1,18]
169	Zimbabwe	0	[0,50]

Table 7: Sensitivity indices for the average absolute shift in rank

Input factor	Main effect	Total effects	Interactions
All			
X_1	0.00	0.07	0.07
X_2	0.00	0.07	0.07
X_{2a}	0.00	0.05	0.05
X_3	0.01	0.07	0.06
X_{3a}	0.01	0.06	0.05
X_4	0.04	0.24	0.20
X_5	0.27	0.60	0.33
X_6	0.18	0.59	0.42
Very high HD			
X_1	0.00	0.05	0.05
X_2	0.00	0.04	0.04
X_{2a}	0.00	0.04	0.04
X_3	0.00	0.05	0.05
X_{3a}	0.00	0.05	0.05
X_4	0.01	0.10	0.09
X_5	0.05	0.53	0.47
X_6	0.43	0.92	0.50
High HD			
X_1	0.00	0.08	0.08
X_2	0.00	0.06	0.06
X_{2a}	0.00	0.05	0.05
X_3	0.00	0.07	0.06
X_{3a}	0.01	0.06	0.05
X_4	0.02	0.20	0.18
X_5	0.40	0.70	0.30
X_6	0.12	0.51	0.38
Medium HD			
X_1	0.00	0.12	0.12
X_2	0.00	0.12	0.12
X_{2a}	0.01	0.09	0.09
X_3	0.00	0.07	0.06
X_{3a}	0.00	0.06	0.06
X_6	0.05	0.37	0.33
X_5	0.17	0.45	0.28
X_6	0.28	0.59	0.31
Low HD			
X_1	0.00	0.07	0.07
X_2	0.01	0.15	0.14
X_{2a}	0.01	0.13	0.12
X_3	0.03	0.16	0.13
X_{3a}	0.02	0.12	0.10
X_4	0.05	0.31	0.26
X_5	0.21	0.50	0.29
X_6	0.12	0.50	0.38

Figure 17: Distribution of the HDI scores when the measurement is $\epsilon \sim N(0, 10\%\sigma)$

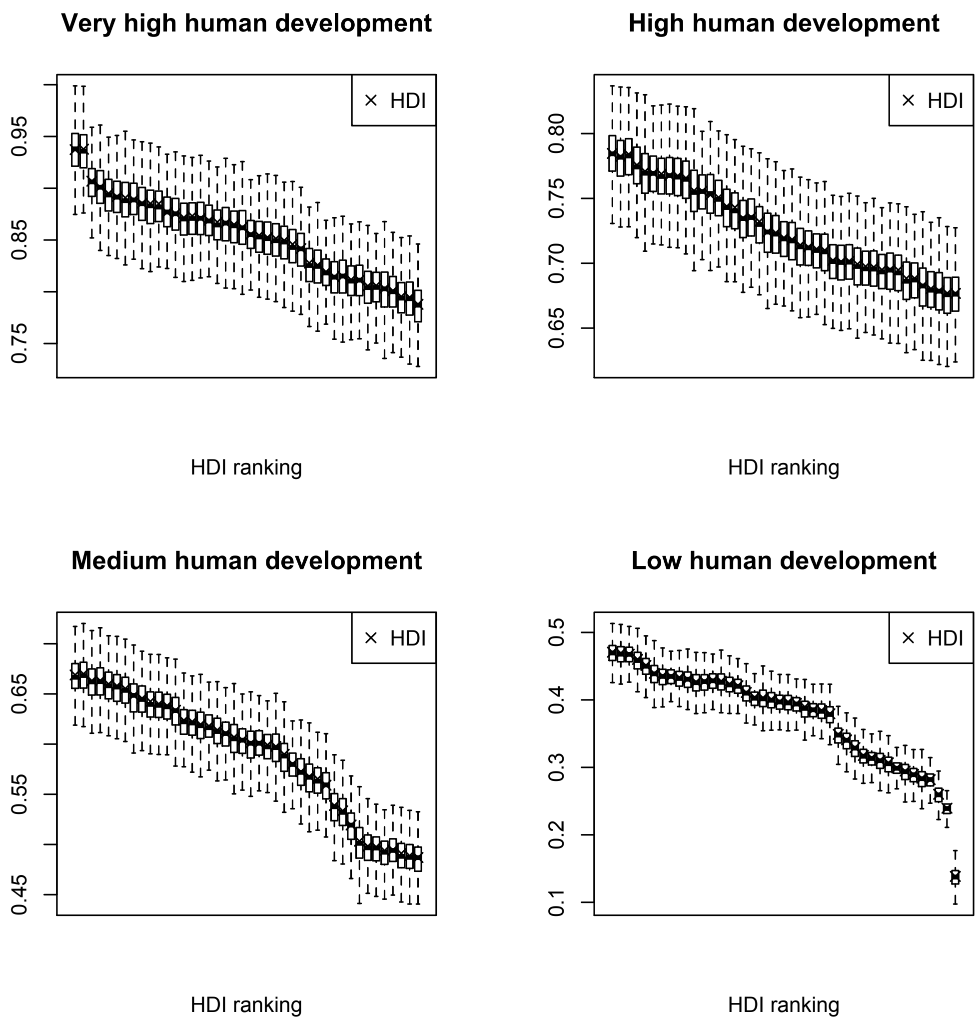


Figure 18: Distribution of the HDI ranking shift when the measurement is $\epsilon \sim N(0, 20\%\sigma)$

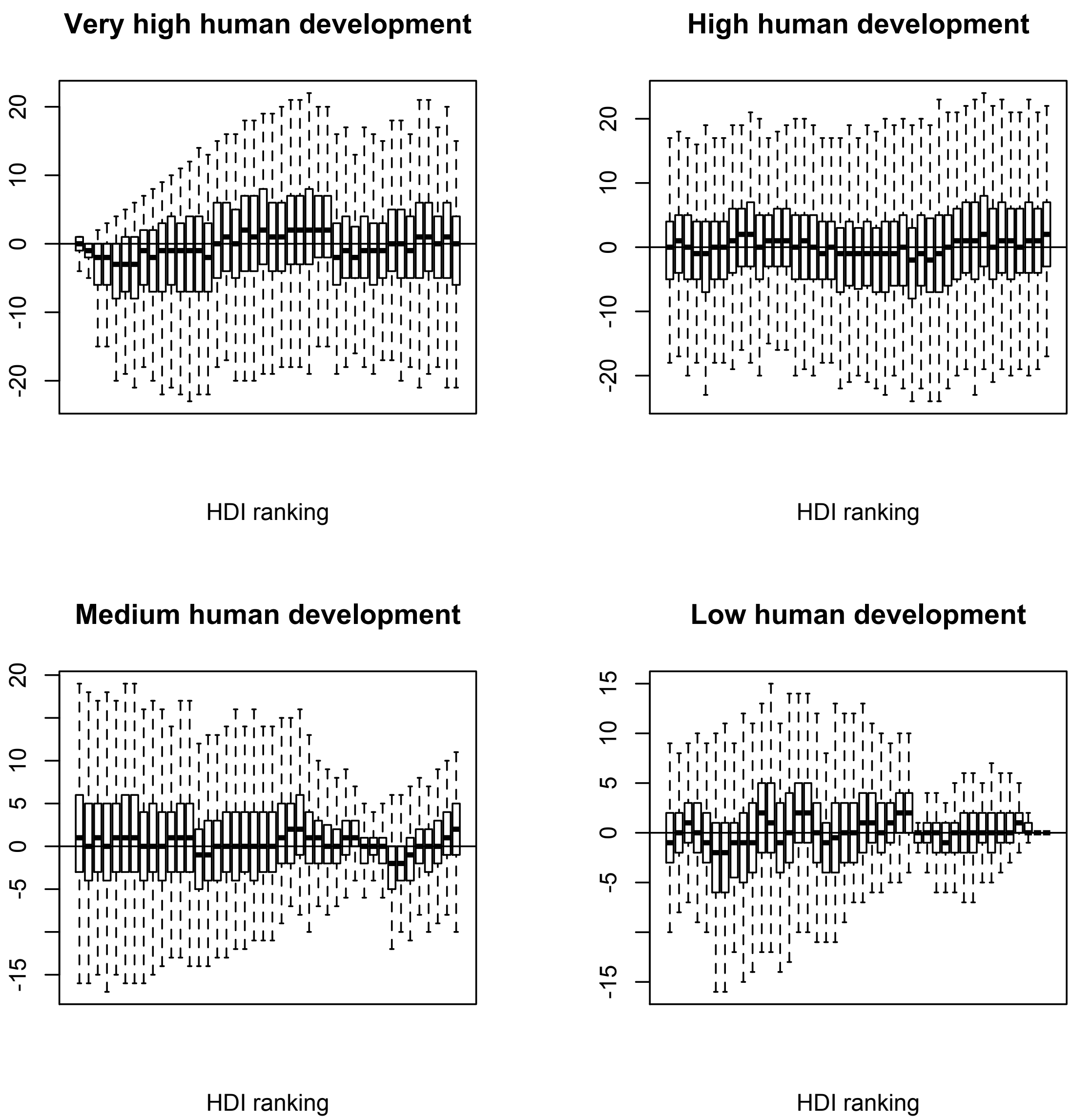
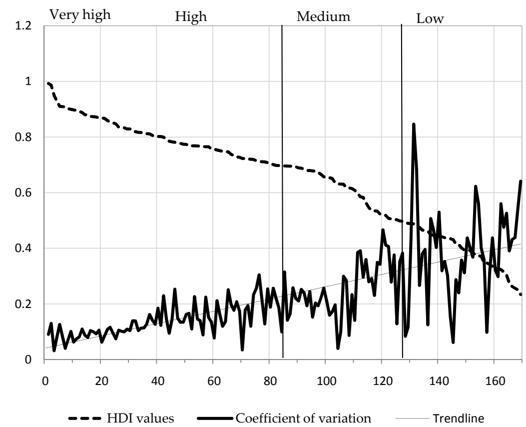


Figure 19: HDI using an arithmetic aggregation (ordered by the associated HDI ranking)



Measures of association between clusters:

- single: distance between the closest members of the two clusters.
- complete: distance between the farthest apart members
- average: distances between all pairs and averages all of these distances.
- median: distances between all pairs and median all of these distances.
- centroid: finding the mean vector location for each of the clusters and taking the distance between these two centroids.
- Ward: based on analysis of variance: maximize r^2.

Figure 20: Different measures of association between clusters

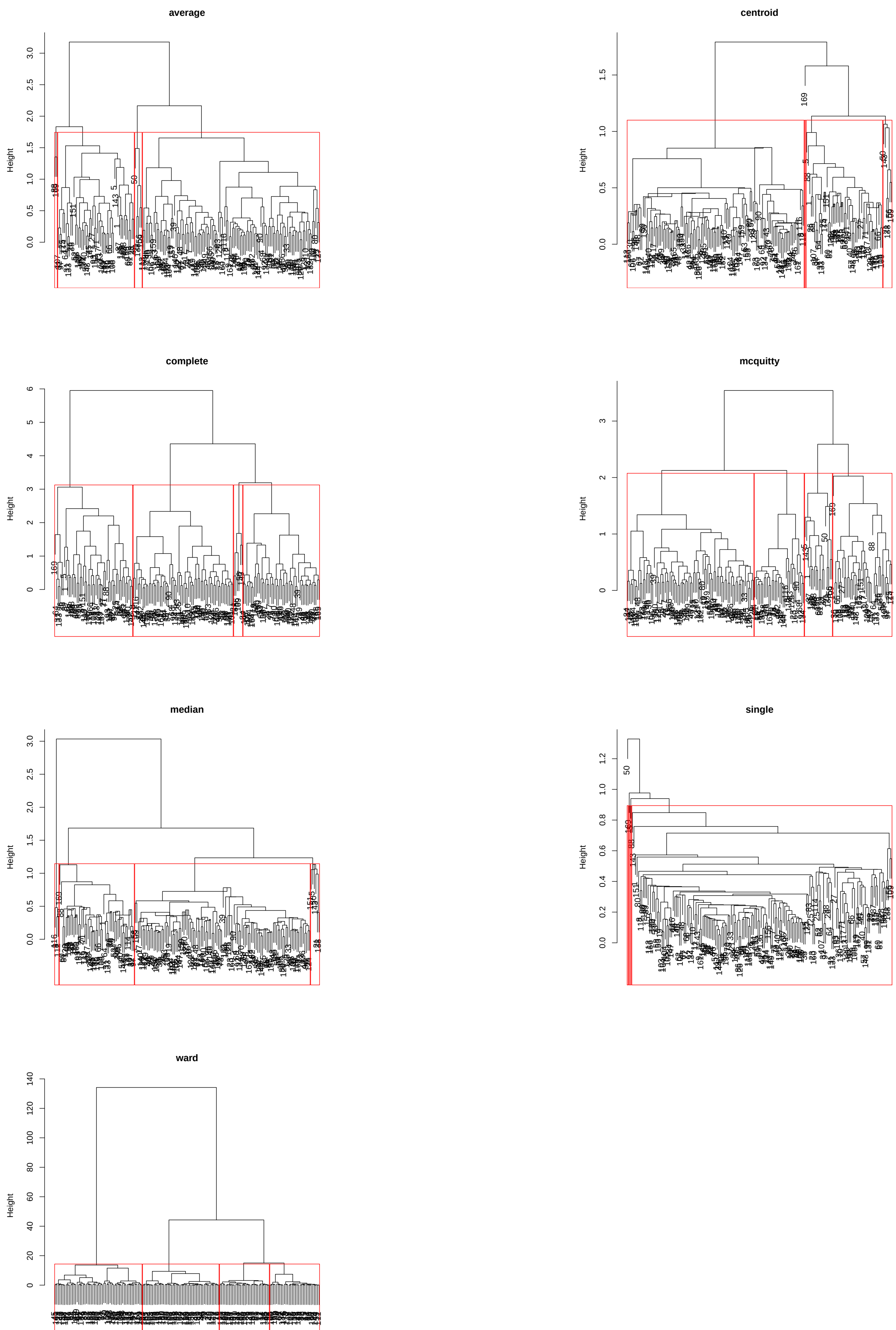


Table 8: Hierarchical clustering using the Ward method over the HDI dimensions and the HDI

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Afghanistan	1	0.35	Albania	3	0.72	Andorra	4	0.82	Angola	1	0.4
Burkina Faso	1	0.3	Algeria	3	0.68	Argentina	3	0.78	Bangladesh	1	0.47
Burundi	1	0.28	Armenia	3	0.7	Australia	4	0.94	Benin	1	0.44
Central African Rep.	1	0.32	Azerbaijan	3	0.71	Austria	4	0.85	Cambodia	2	0.49
Chad	1	0.3	Belarus	3	0.73	Bahamas	3	0.78	Cameroon	1	0.46
Congo (D. R.)	1	0.24	Belize	3	0.69	Bahrain	4	0.8	Comoros	1	0.43
Ethiopia	1	0.33	Bolivia	2	0.64	Barbados	4	0.79	Congo	2	0.49
Guinea	1	0.34	Bosnia & Herzeg.	3	0.71	Belgium	4	0.87	Côte d'Ivoire	1	0.4
Guinea-Bissau	1	0.29	Botswana	2	0.63	Brunei Darussalam	4	0.8	Djibouti	1	0.4
Mali	1	0.31	Brazil	3	0.7	Canada	4	0.89	Gambia	1	0.39
Mozambique	1	0.28	Bulgaria	3	0.74	Chile	3	0.78	Ghana	1	0.47
Niger	1	0.26	Cape Verde	2	0.53	Croatia	3	0.77	Haiti	1	0.4
Sierra Leone	1	0.32	China	2	0.66	Cyprus	4	0.81	India	2	0.52
Zimbabwe	1	0.14	Colombia	3	0.69	Czech Republic	4	0.84	Kenya	1	0.47
			Costa Rica	3	0.72	Denmark	4	0.87	Lao P.D.R.	2	0.5
			Dominican Rep.	2	0.66	Estonia	4	0.81	Lesotho	1	0.43
			Ecuador	3	0.7	Finland	4	0.87	Liberia	1	0.3
			Egypt	2	0.62	France	4	0.87	Madagascar	1	0.44
			El Salvador	2	0.66	Germany	4	0.88	Malawi	1	0.38
			Equatorial Guinea	2	0.54	Greece	4	0.86	Mauritania	1	0.43
			Fiji	2	0.67	Hong Kong, China	4	0.86	Myanmar	1	0.45
			Gabon	2	0.65	Hungary	4	0.8	Nepal	1	0.43
			Georgia	3	0.7	Iceland	4	0.87	Nigeria	1	0.42
			Guatemala	2	0.56	Ireland	4	0.9	Pakistan	2	0.49
			Guyana	2	0.61	Israel	4	0.87	Papua New Guinea	1	0.43
			Honduras	2	0.6	Italy	4	0.85	Rwanda	1	0.38
			Indonesia	2	0.6	Japan	4	0.88	Sao Tome & Principe	2	0.49
			Iran	3	0.7	Korea	4	0.88	Senegal	1	0.41
			Jamaica	3	0.69	Kuwait	3	0.77	Solomon Islands	2	0.49
			Jordan	3	0.68	Latvia	3	0.77	Sudan	1	0.38
			Kazakhstan	3	0.71	Libya	3	0.76	Swaziland	2	0.5
			Kyrgyzstan	2	0.6	Liechtenstein	4	0.89	Tanzania	1	0.4
			Maldives	2	0.6	Lithuania	3	0.78	Timor-Leste	2	0.5
			Mauritius	3	0.7	Luxembourg	4	0.85	Togo	1	0.43
			Micronesia	2	0.61	Malaysia	3	0.74	Uganda	1	0.42
			Moldova	2	0.62	Malta	4	0.82	Yemen	1	0.44
			Mongolia	2	0.62	Mexico	3	0.75	Zambia	1	0.4
			Morocco	2	0.57	Montenegro	3	0.77			
			Namibia	2	0.61	Netherlands	4	0.89			
			Nicaragua	2	0.56	New Zealand	4	0.91			
			Paraguay	2	0.64	Norway	4	0.94			

Table 8: Hierarchical clustering using the Ward method over the HDI dimensions and the HDI

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
			Peru	3	0.72	Panama	3	0.76			
			Philippines	2	0.64	Poland	4	0.8			
			Russia	3	0.72	Portugal	4	0.8			
			Serbia	3	0.74	Qatar	4	0.8			
			South Africa	2	0.6	Romania	3	0.77			
			Sri Lanka	2	0.66	Saudi Arabia	3	0.75			
			Suriname	2	0.65	Singapore	4	0.85			
			Syria	2	0.59	Slovakia	4	0.82			
			Tajikistan	2	0.58	Slovenia	4	0.83			
			Thailand	2	0.65	Spain	4	0.86			
			Macedonia	3	0.7	Sweden	4	0.88			
			Tonga	3	0.68	Switzerland	4	0.87			
			Tunisia	3	0.68	Trinidad & Tobago	3	0.74			
			Turkey	3	0.68	U. A. E.	4	0.82			
			Turkmenistan	2	0.67	United Kingdom	4	0.85			
			Ukraine	3	0.71	United States	4	0.9			
			Uzbekistan	2	0.62	Uruguay	3	0.76			
			Venezuela	3	0.7						
			Viet Nam	2	0.57						

Table 9: Hierarchical clustering using the Ward method over the HDI indicators

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Afghanistan	1	0.35	Albania	3	0.72	Australia	4	0.94	Algeria	3	0.68
Angola	1	0.4	Andorra	4	0.82	Austria	4	0.85	Brazil	3	0.7
Bangladesh	1	0.47	Argentina	3	0.78	Belgium	4	0.87	Cape Verde	2	0.53
Benin	1	0.44	Armenia	3	0.7	Brunei Darussalam	4	0.8	China	2	0.66
Botswana	2	0.63	Azerbaijan	3	0.71	Canada	4	0.89	Colombia	3	0.69
Burkina Faso	1	0.3	Bahamas	3	0.78	Denmark	4	0.87	Dominican Rep.	2	0.66
Burundi	1	0.28	Bahrain	4	0.8	Finland	4	0.87	Ecuador	3	0.7
Cambodia	2	0.49	Barbados	4	0.79	France	4	0.87	Egypt	2	0.62
Cameroon	1	0.46	Belarus	3	0.73	Germany	4	0.88	El Salvador	2	0.66
Central African Rep.	1	0.32	Belize	3	0.69	Greece	4	0.86	Guatemala	2	0.56
Chad	1	0.3	Bolivia	2	0.64	Hong Kong, China	4	0.86	Honduras	2	0.6
Comoros	1	0.43	Bosnia & Herzeg.	3	0.71	Iceland	4	0.87	Indonesia	2	0.6
Congo	2	0.49	Bulgaria	3	0.74	Ireland	4	0.9	Iran	3	0.7
D. R. Congo	1	0.24	Chile	3	0.78	Israel	4	0.87	Maldives	2	0.6
Côte d'Ivoire	1	0.4	Costa Rica	3	0.72	Italy	4	0.85	Mauritius	3	0.7
Djibouti	1	0.4	Croatia	3	0.77	Japan	4	0.88	Morocco	2	0.57
Equatorial Guinea	2	0.54	Cyprus	4	0.81	Korea	4	0.88	Nicaragua	2	0.56

Table 9: Hierarchical clustering using the Ward method over the HDI indicators

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Ethiopia	1	0.33	Czech Republic	4	0.84	Kuwait	3	0.77	Paraguay	2	0.64
Gabon	2	0.65	Estonia	4	0.81	Liechtenstein	4	0.89	Suriname	2	0.65
Gambia	1	0.39	Fiji	2	0.67	Luxembourg	4	0.85	Syria	2	0.59
Ghana	1	0.47	Georgia	3	0.7	Netherlands	4	0.89	Thailand	2	0.65
Guinea	1	0.34	Guyana	2	0.61	New Zealand	4	0.91	Tunisia	3	0.68
Guinea-Bissau	1	0.29	Hungary	4	0.8	Norway	4	0.94	Turkey	3	0.68
Haiti	1	0.4	Jamaica	3	0.69	Qatar	4	0.8	Venezuela	3	0.7
India	2	0.52	Jordan	3	0.68	Singapore	4	0.85	Viet Nam	2	0.57
Kenya	1	0.47	Kazakhstan	3	0.71	Spain	4	0.86			
Lao P.D.R.	2	0.5	Kyrgyzstan	2	0.6	Sweden	4	0.88			
Lesotho	1	0.43	Latvia	3	0.77	Switzerland	4	0.87			
Liberia	1	0.3	Libya	3	0.76	U. A. E.	4	0.82			
Madagascar	1	0.44	Lithuania	3	0.78	United Kingdom	4	0.85			
Malawi	1	0.38	Malaysia	3	0.74	United States	4	0.9			
Mali	1	0.31	Malta	4	0.82						
Mauritania	1	0.43	Mexico	3	0.75						
Mozambique	1	0.28	Micronesia	2	0.61						
Myanmar	1	0.45	Moldova	2	0.62						
Namibia	2	0.61	Mongolia	2	0.62						
Nepal	1	0.43	Montenegro	3	0.77						
Niger	1	0.26	Panama	3	0.76						
Nigeria	1	0.42	Peru	3	0.72						
Pakistan	2	0.49	Philippines	2	0.64						
Papua New Guinea	1	0.43	Poland	4	0.8						
Rwanda	1	0.38	Portugal	4	0.8						
Sao Tome & Principe	2	0.49	Romania	3	0.77						
Senegal	1	0.41	Russia	3	0.72						
Sierra Leone	1	0.32	Saudi Arabia	3	0.75						
Solomon Islands	2	0.49	Serbia	3	0.74						
South Africa	2	0.6	Slovakia	4	0.82						
Sudan	1	0.38	Slovenia	4	0.83						
Swaziland	2	0.5	Sri Lanka	2	0.66						
Tanzania	1	0.4	Tajikistan	2	0.58						
Timor-Leste	2	0.5	Macedonia	3	0.7						
Togo	1	0.43	Tonga	3	0.68						
Uganda	1	0.42	Trinidad & Tobago	3	0.74						
Yemen	1	0.44	Turkmenistan	2	0.67						
Zambia	1	0.4	Ukraine	3	0.71						
Zimbabwe	1	0.14	Uruguay	3	0.76						
			Uzbekistan	2	0.62						

Table 10: Hierarchical clustering using the Ward method over the HDI indicators and the HDI

Cluster 1	DG[29]	HDI	Cluster 2	DG	HDI	Cluster 3	DG	HDI	Cluster 4	DG	HDI
Afghanistan	1	0.35	Albania	3	0.72	Andorra	4	0.82	Argentina	3	0.78
Angola	1	0.4	Algeria	3	0.68	Australia	4	0.94	Bahamas	3	0.78
Bangladesh	1	0.47	Armenia	3	0.7	Austria	4	0.85	Bahrain	4	0.8
Benin	1	0.44	Azerbaijan	3	0.71	Belgium	4	0.87	Barbados	4	0.79
Burkina Faso	1	0.3	Belarus	3	0.73	Brunei Darussalam	4	0.8	Chile	3	0.78
Burundi	1	0.28	Belize	3	0.69	Canada	4	0.89	Cyprus	4	0.81
Cambodia	2	0.49	Bolivia	2	0.64	Denmark	4	0.87	Czech Republic	4	0.84
Cameroon	1	0.46	Bosnia & Herzeg.	3	0.71	Finland	4	0.87	Estonia	4	0.81
Central African Rep.	1	0.32	Botswana	2	0.63	France	4	0.87	Hungary	4	0.8
Chad	1	0.3	Brazil	3	0.7	Germany	4	0.88	Latvia	3	0.77
Comoros	1	0.43	Bulgaria	3	0.74	Greece	4	0.86	Libya	3	0.76
Congo	2	0.49	Cape Verde	2	0.53	Hong Kong, China	4	0.86	Lithuania	3	0.78
D. R. Congo	1	0.24	China	2	0.66	Iceland	4	0.87	Malta	4	0.82
Côte d'Ivoire	1	0.4	Colombia	3	0.69	Ireland	4	0.9	Montenegro	3	0.77
Djibouti	1	0.4	Costa Rica	3	0.72	Israel	4	0.87	Poland	4	0.8
Equatorial Guinea	2	0.54	Croatia	3	0.77	Italy	4	0.85	Portugal	4	0.8
Ethiopia	1	0.33	Dominican Republic	2	0.66	Japan	4	0.88	Romania	3	0.77
Gambia	1	0.39	Ecuador	3	0.7	Korea	4	0.88	Saudi Arabia	3	0.75
Ghana	1	0.47	Egypt	2	0.62	Kuwait	3	0.77	Slovakia	4	0.82
Guinea	1	0.34	El Salvador	2	0.66	Liechtenstein	4	0.89	Slovenia	4	0.83
Guinea-Bissau	1	0.29	Fiji	2	0.67	Luxembourg	4	0.85	Trinidad & Tobago	3	0.74
Haiti	1	0.4	Gabon	2	0.65	Netherlands	4	0.89	Uruguay	3	0.76
India	2	0.52	Georgia	3	0.7	New Zealand	4	0.91			
Kenya	1	0.47	Guatemala	2	0.56	Norway	4	0.94			
Lao P.D.R.	2	0.5	Guyana	2	0.61	Qatar	4	0.8			
Lesotho	1	0.43	Honduras	2	0.6	Singapore	4	0.85			
Liberia	1	0.3	Indonesia	2	0.6	Spain	4	0.86			
Madagascar	1	0.44	Iran	3	0.7	Sweden	4	0.88			
Malawi	1	0.38	Jamaica	3	0.69	Switzerland	4	0.87			
Mali	1	0.31	Jordan	3	0.68	U. A. E.	4	0.82			
Mauritania	1	0.43	Kazakhstan	3	0.71	United Kingdom	4	0.85			
Mozambique	1	0.28	Kyrgyzstan	2	0.6	United States	4	0.9			
Myanmar	1	0.45	Malaysia	3	0.74						
Nepal	1	0.43	Maldives	2	0.6						
Niger	1	0.26	Mauritius	3	0.7						
Nigeria	1	0.42	Mexico	3	0.75						
Pakistan	2	0.49	Micronesia	2	0.61						
Papua New Guinea	1	0.43	Moldova	2	0.62						
Rwanda	1	0.38	Mongolia	2	0.62						
Sao Tome & Principe	2	0.49	Morocco	2	0.57						
Senegal	1	0.41	Namibia	2	0.61						

Sierra Leone	1	0.32	Nicaragua	2	0.56
Solomon Islands	2	0.49	Panama	3	0.76
Sudan	1	0.38	Paraguay	2	0.64
Swaziland	2	0.5	Peru	3	0.72
Tanzania	1	0.4	Philippines	2	0.64
Timor-Leste	2	0.5	Russia	3	0.72
Togo	1	0.43	Serbia	3	0.74
Uganda	1	0.42	South Africa	2	0.6
Yemen	1	0.44	Sri Lanka	2	0.66
Zambia	1	0.4	Suriname	2	0.65
Zimbabwe	1	0.14	Syria	2	0.59
			Tajikistan	2	0.58
			Thailand	2	0.65
			Macedonia	3	0.7
			Tonga	3	0.68
			Tunisia	3	0.68
			Turkey	3	0.68
			Turkmenistan	2	0.67
			Ukraine	3	0.71
			Uzbekistan	2	0.62
			Venezuela	3	0.7
			Viet Nam	2	0.57

Figure 21: Non-hierarchical clustering: k-means method